Testing the mezofractal power spectrum for ISM by comparing with numerical MHD calculations

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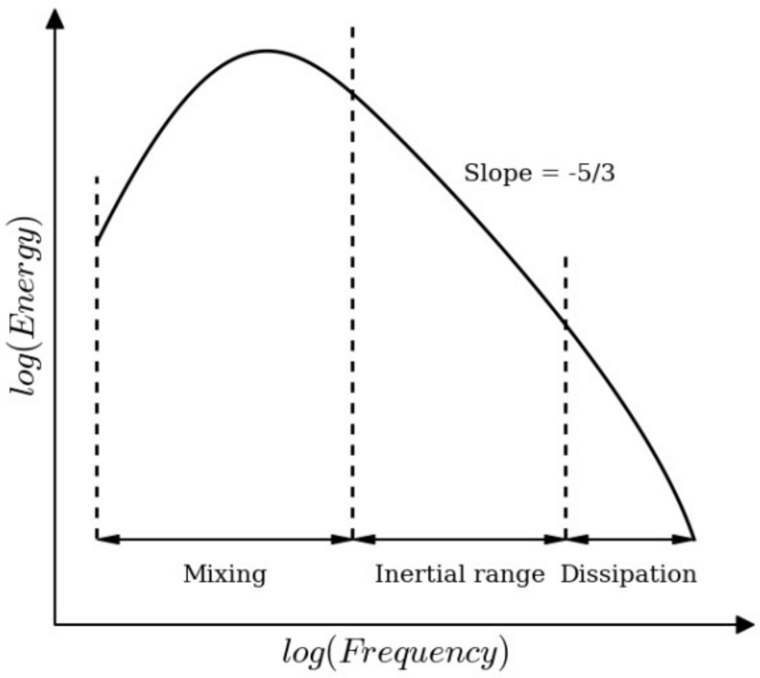
Kolmogorov spectrum

The hypotheses concerning the local structure of turbulence at high Reynolds number were based physically on Richardson's idea of the existence in the turbulent flow of vortices on all possible scales $l < r < L$ between the 'external scale' L and the 'internal scale' l and of a certain uniform mechanism of energy transfer from the coarser-scaled vortices to the finer.

A. N. Kolmogorov. “A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number”. B: Journal of Fluid Mechanics 13.1 (1962), c. 82—85.



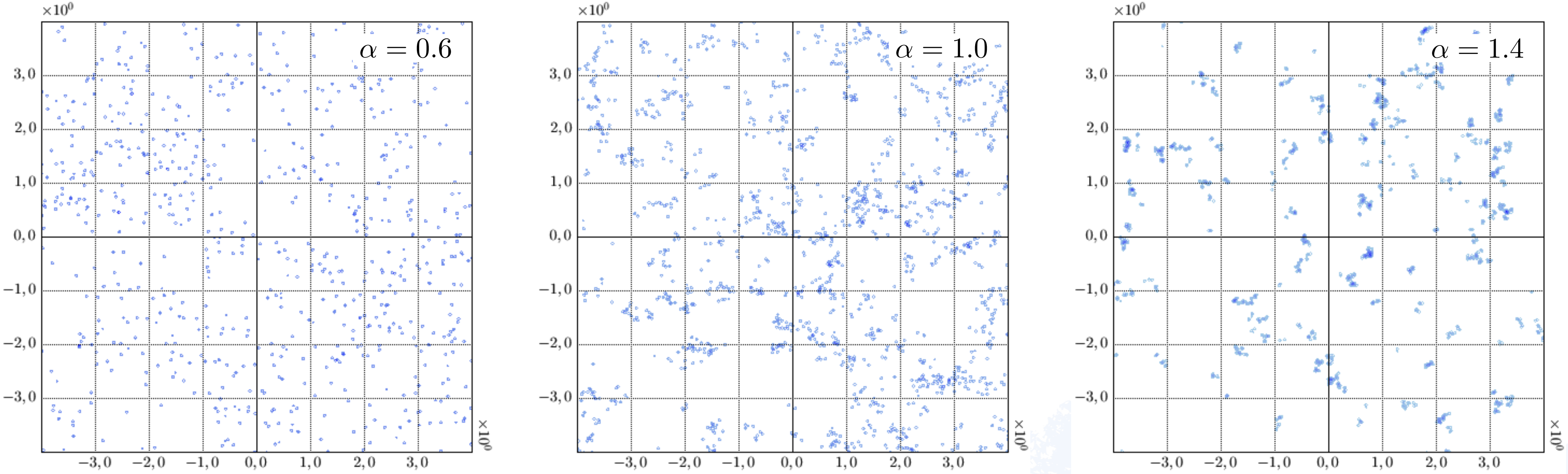
Mesofractal modelling

Моделирование методом Монте-Карло мезофрактальной системы с заданными параметрами включает следующие этапы:

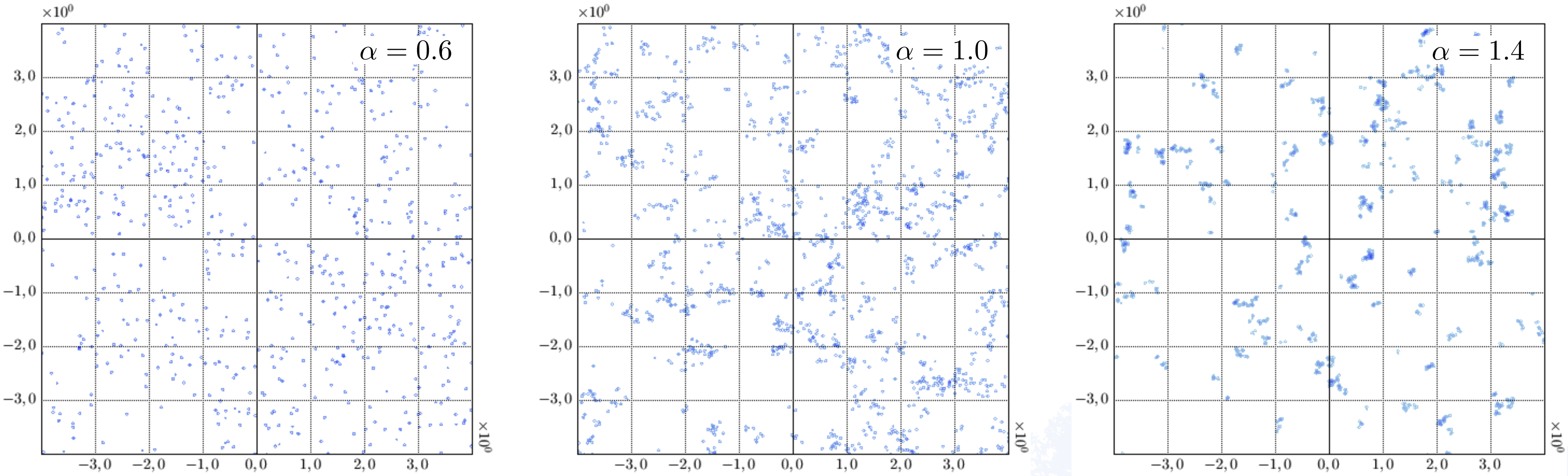
1) In three-dimensional space, a homogeneous Poisson ensemble of random points is constructed;
2) Each of them is taken as the initial node for the implementation of the three-dimensional Markov chain with the characteristic function of the random transition vector $\exp\{-(bk)^{\alpha}\}$ corresponding to the three-dimensional isotropic Lévy–Feldheim distribution;
3) With probability *1 - c* the trajectory terminates at this point; otherwise, the point continues its movement according to the same rules.

As a result, we get a set of correlated points with average concentration $n = n_0/(1\text{-}c)$, where n_0 is the concentration of seed nodes.

Mesofractal modelling



Mesofractal modelling



Poisson

Mesofractal

Fractal

Uchaikin-Zolotarev Power spectrum

$$P_{UZ}(k) = A\frac{e^{-(bk)^{\alpha}}}{1 - ce^{-(bk)^{\alpha}}}$$

where A is a normalizing constant, k is the modulus of the wave vector, b is the length scale, $c \in (0,1]$ and $\alpha \in (0,2]$.

The exponent $\exp\{-(bk)^{\alpha}\}$ is the characteristic function of Lévy–Feldheim 3D isotropic stable distribution that describes the random run of a Markov chain. In this case, the expression itself satisfies the Orstein-Zernike equation:

$$\xi(\mathbf{x}) = Ab^{-3}p(\mathbf{x}/b;\alpha) + cb^{-3}\int p(\mathbf{x}/b)\xi(\mathbf{x}-\mathbf{x}')d\mathbf{x}',$$

where $p(\mathbf{x};\alpha)$ is a three-dimensional stable Lévy–Feldheim distribution.

MHD model

Third order of accuracy of compressible magnetohydrodynamic (MHD) turbulence using a hybrid essentially non-oscillatory (ENO) for solving ideal isothermal MHD equations in a periodic box:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0,$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \rho^{-1} \nabla (a^2 \rho) - (\nabla \times \mathbf{B}) \times \mathbf{B}/4\pi\rho = f,$$

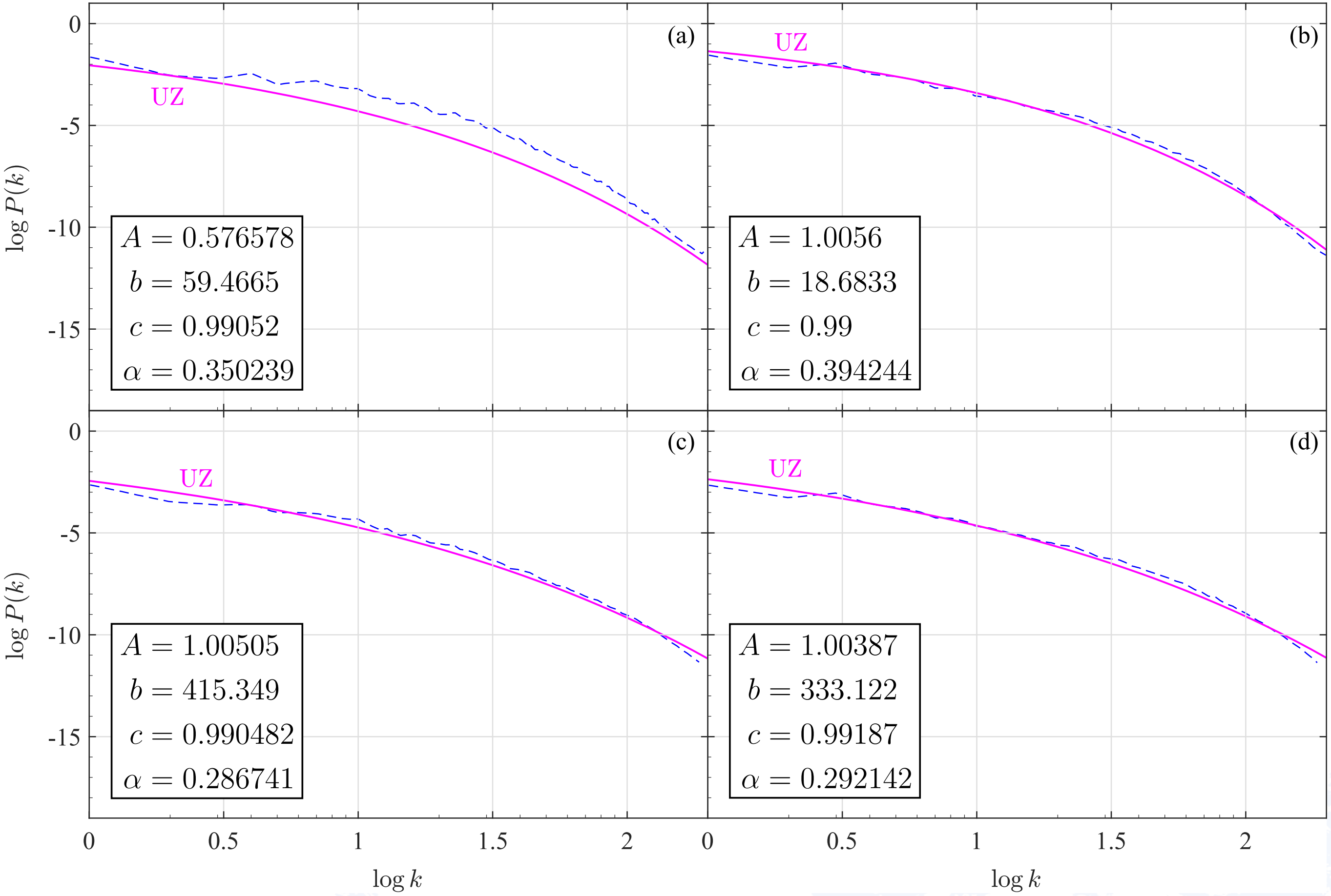
$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

[Cho, J.; Lazarian, A. Compressible magnetohydrodynamic turbulence: mode coupling, scaling relations, anisotropy, viscositydamped regime and astrophysical implications. Mon. Not. R. Astron. Soc. 2003, 345, 325–339.]

Approximation

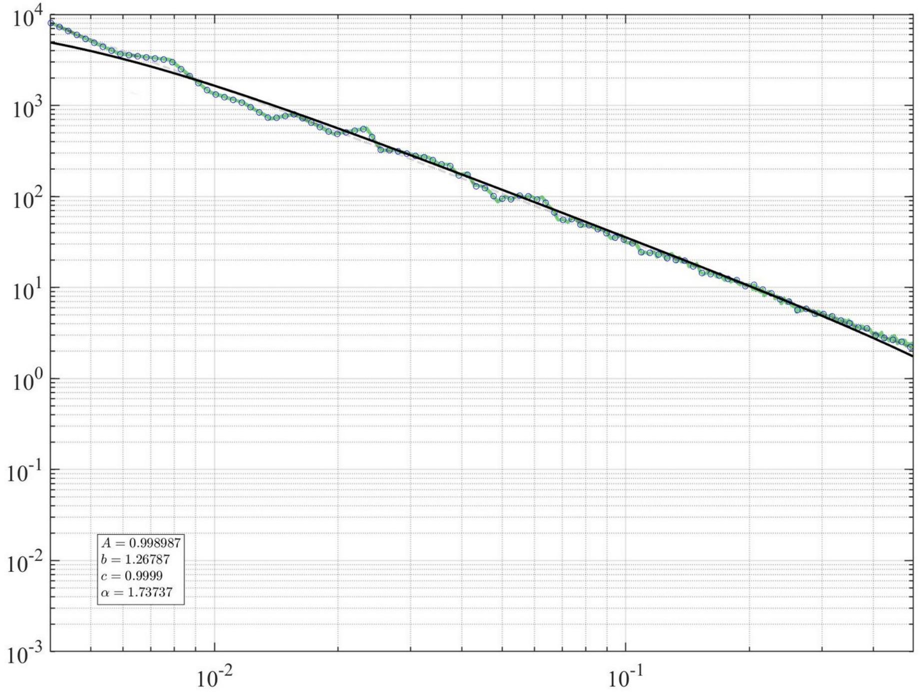
$$\text{MSLE} = \frac{1}{n}\sum_i (\ln P(\hat{k}_i; A, b, c, \alpha) - \ln \hat{P}_i)^2,$$

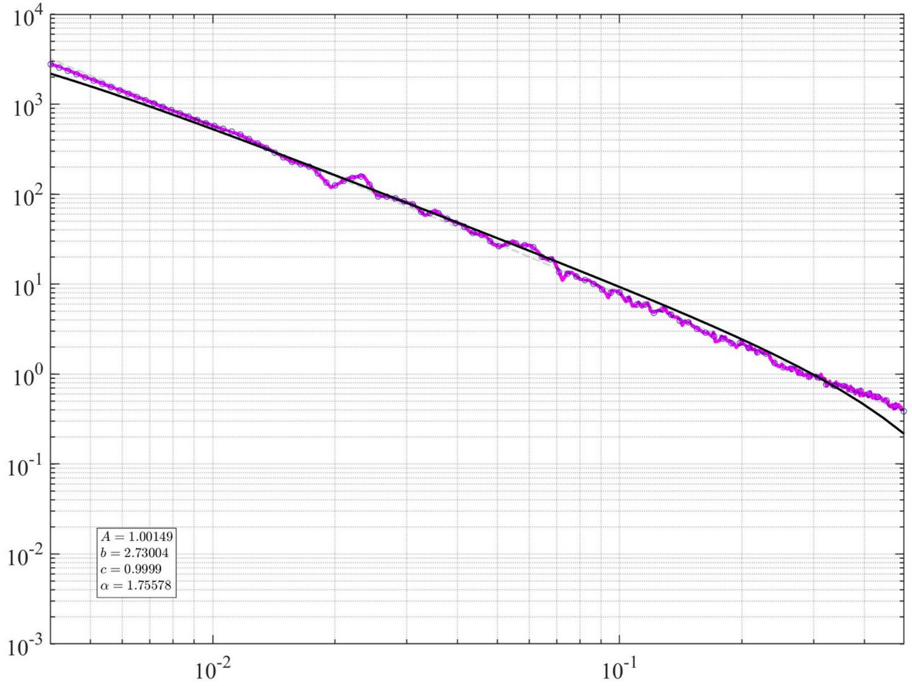
where $\widehat{k_i}$ is the experimental value of the wave vector and $\widehat{P_i}$ is the experimental value of the power spectrum.



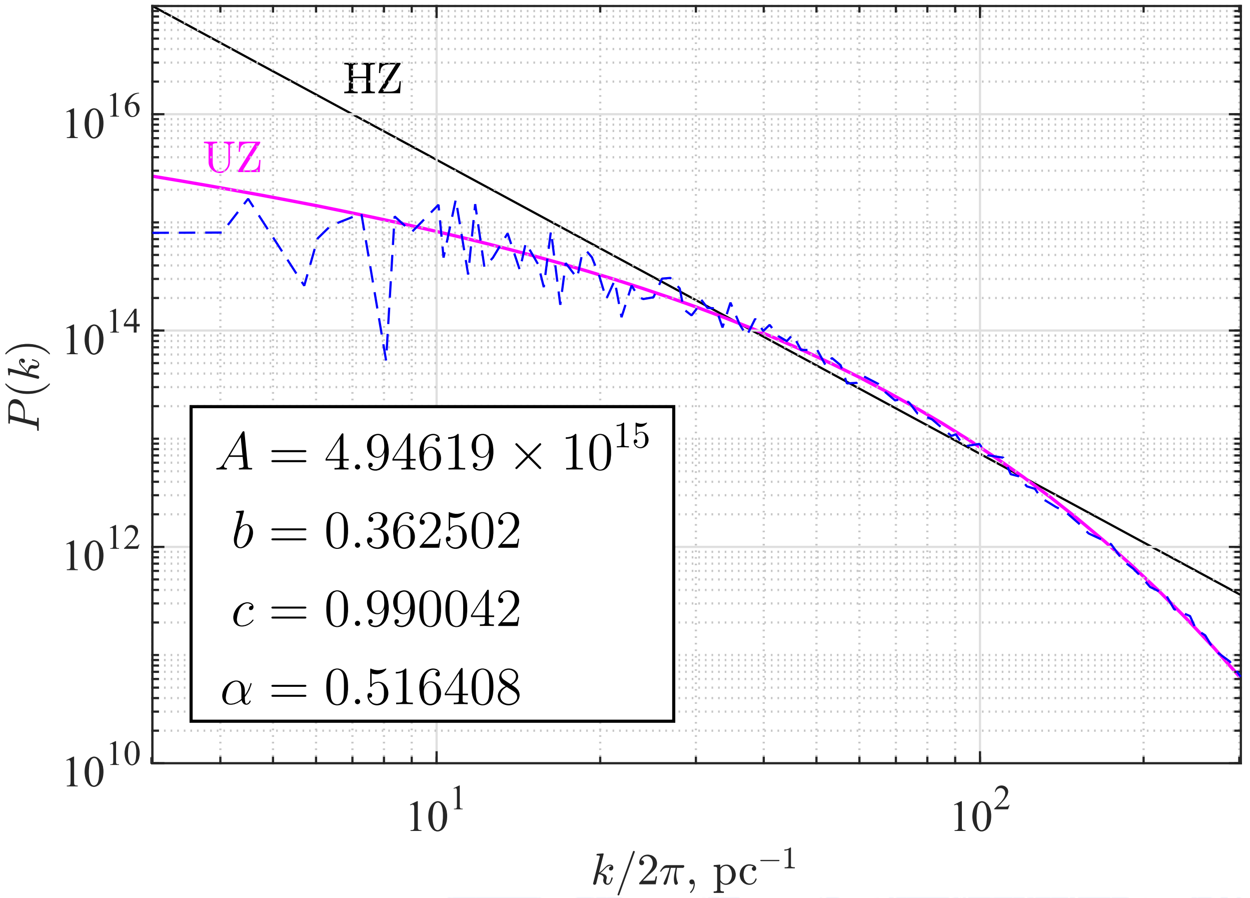
A) is the energy spectrum generated MHD simulation with a lower Alfvénic-Mach number and the thick case of varying abundances;
B) is the same with a greater Alfvénic-Mach number and the thick case of varying abundances;
C) is the same with a lower Alfvénic-Mach number and the normal case of varying abundances;
D) is the same with a greater Alfvénic-Mach number and the normal case of varying abundances.

[Burkhart, B.; Lazarian, A.; Ossenkopf, V.; Stutzki, J. The turbulence power spectrum in optically thick interstellar clouds. The Astrophysical Journal 2013, 771, 123. doi:10.1088/0004-637X/771/2/123.]

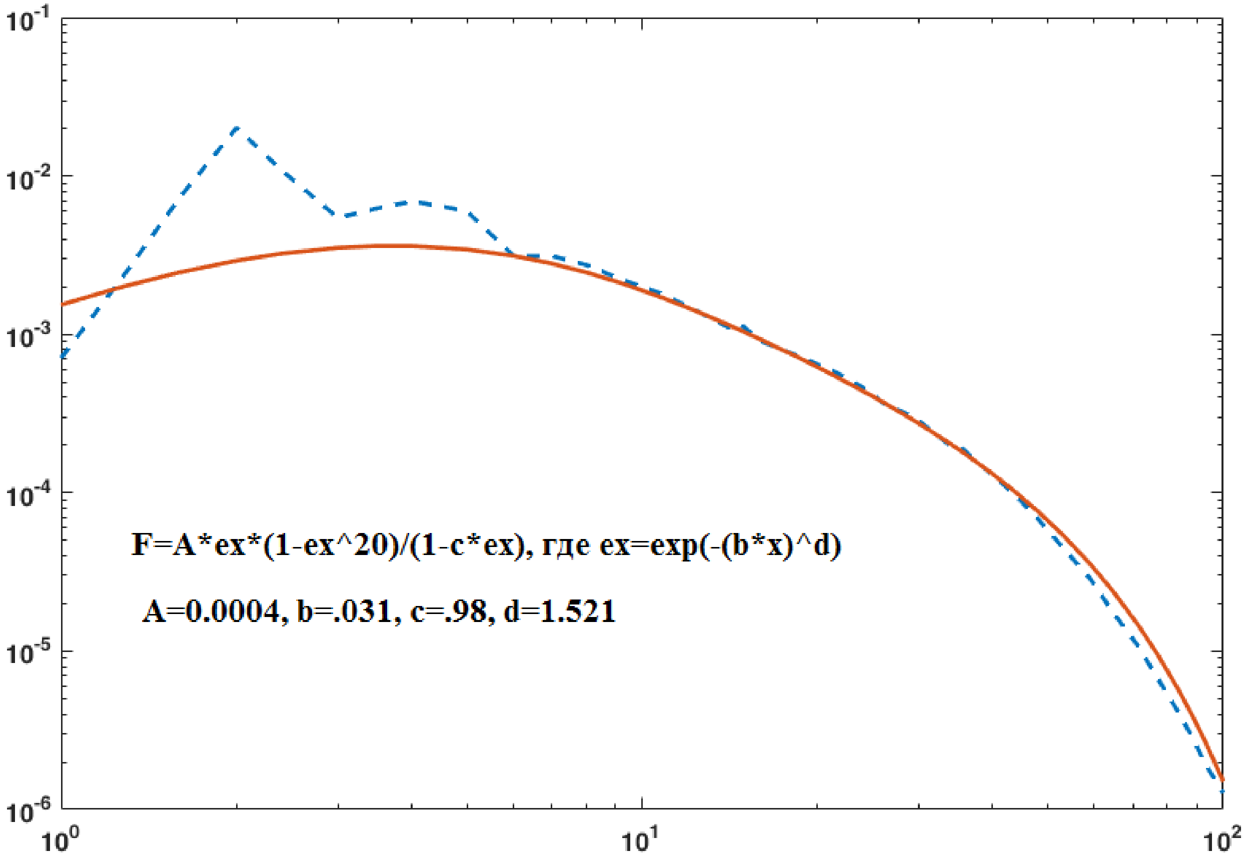




[Robitaille, J.-F. и et. al. “Statistical model for filamentary structures of molecular clouds - The modified multiplicative random cascade model and its multifractal nature”. В: A&A 641 (2020), A138.]



[Inoue, T.; Yamazaki, R.; Inutsuka, S.i. Turbulence and magnetic field amplification in supernova remnants: Interactions between a strong shock wave and multiphase interstellar medium. The Astrophysical Journal 2009, 695, 825.]



[D. Falceta-Gonçalves et. al. “Turbulence in the interstellar medium”. В: Nonlinear Processes in Geophysics 21.3 (may 2014), с. 587—604.]

Further work

- Calculate of errors of approximations on different parts of power spectrum.
- Analyze correlations between parameters of UZ power spectrum and other statistical properties of ISM.

Thank you for your attention!