Application of digital processing of time series of muonograms for the analysis of extreme events in the heliosphere

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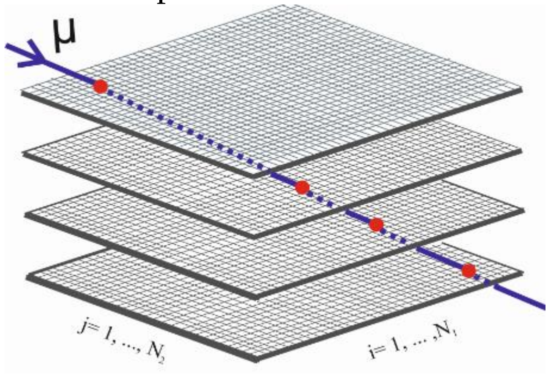
1. URAGAN muon hodoscope (MH), muonograms , matrix MH data

URAGAN muon hodoscope



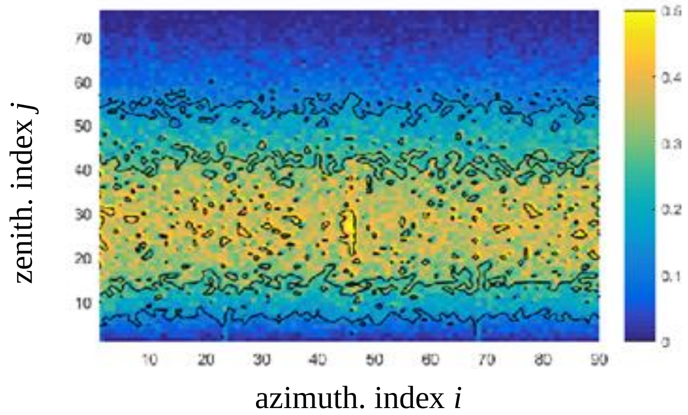
$\{i_1, j_1, \ldots, i_8, j_8\} \Rightarrow (\varphi, \theta)$

Azimuthal angle $\varphi_i = \Delta\varphi(i-1)$, $\Delta\varphi = 4^\circ$, $0 \le \varphi < 360^\circ$
Zenithal angle $\theta_j = \Delta\theta(j-1)$, $\Delta\theta = 1^\circ$, $0 \le \theta < 76^\circ$

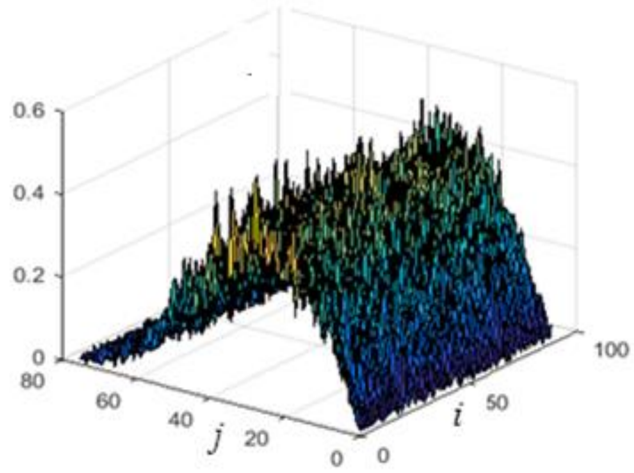
Matrix MH data $Y(i, j, Tk)$ - the number of muons registered within a solid angle (φ_i, θ_j)
during a time period $(T(k-1) \le t < Tk)$, $k = 1, 2, \ldots$

$i = 1, \ldots, N_1$ $j = 1, \ldots, N_2$
$N_1 = 90$, $N_2 = 76$ $T = 1$ minute, $T = 1$ hour

1-minute and 1-hour-sampled normalized MH data



2D muonogram - MH data matrices $Y(i, j, Tk)$



3D muonogram -MH data matrix $Y(i, j, Tk)$

<u>Extreme events in the heliosphere (EEH) –local anisotropy (LA) – Forbush decreases in MH data</u>

2. Key tasks of matrix MH data time series digital processing for EEH analysis

Output time series of MH data matrices $Y(i, j, Tk), k = 1, 2, \ldots$

Input time series of MH data matrices $Y_0(i, j, Tk), k = 1, 2, \ldots$

Hardware function (HF) of the MH $F(i, j, Tk), k = 1, 2, \ldots,$ $F(i, j, Tk) = F_0(i, j) F_W(j, Tk)$

$F_0(i, j)$ - HF- construction, $F_W(j, Tk)$ -HF atmospheric disturbances

$Y(i, j, Tk) = F(i, j, Tk) Y_0(i, j, Tk)$

<u>Key tasks</u>:

1. Spatial-temporal low-pass filtering and spectral analysis of MH matrix data time series

Filtering - $Y(i, j, Tk) \Rightarrow Y_\Phi(i, j, Tk) = ?\ k = 1, 2, \ldots;$ DFT spectra - $Y(i, j, Tk) \Rightarrow C(r), r = 0, 1, \ldots$

2. Direct tasks - estimation of input MH matrix data time series

$Y(i, j, Tk),\ k = 1, 2, \ldots \Rightarrow Y_0^\circ(i, j, Tk)?,\ k = 1, 2, \ldots$

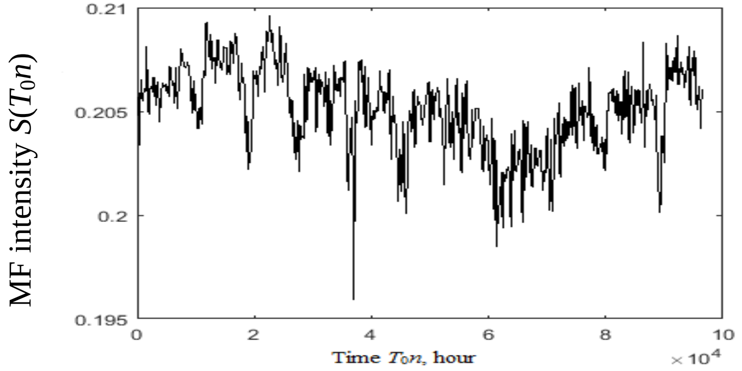
$F^\circ(i, j, Tk) = ?$ $Y_0^\circ(i, j, Tk) = Y(i, j, Tk) / F^\circ(i, j, Tk),\ k = 1, 2, \ldots$ $Y_0^\circ(i, j, Tk) = ?$

3.Indirect tasks - EEH (LA) recognition in matrix data: Forbush decrease recognition

$(i, j) \in \Psi_{a,k} \subset \Psi_0,\ k = 1, 2, \ldots,\ \Psi_0 = \{(i, j): i = 1, \ldots, N_1, j = 1, \ldots, N_2\},\ \Psi_{a,k} \Rightarrow ?$

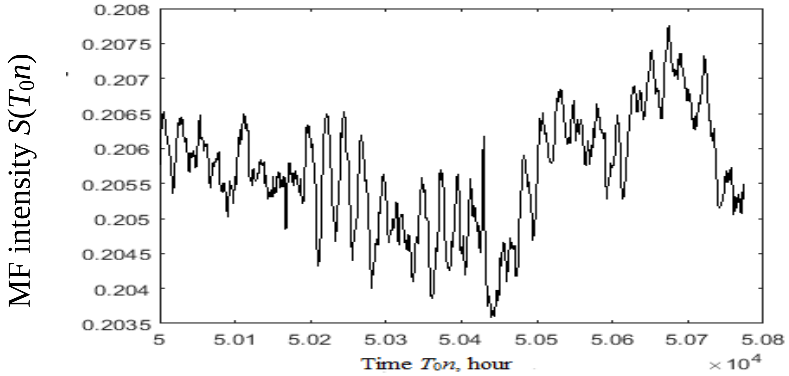
3. MH data spectral analysis

Muon flux (MF) intensity $S(T_0 n) = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} Y(i, j, T_0 n)$



$1 \le n \le N_f$ - 11 years, $1 \Leftrightarrow 01.01.2008$, $N_f \Leftrightarrow 31.12.2018 \Leftrightarrow 96360$

MF intensity $S(T_0 n)$, $n_1 \le n \le n_2$



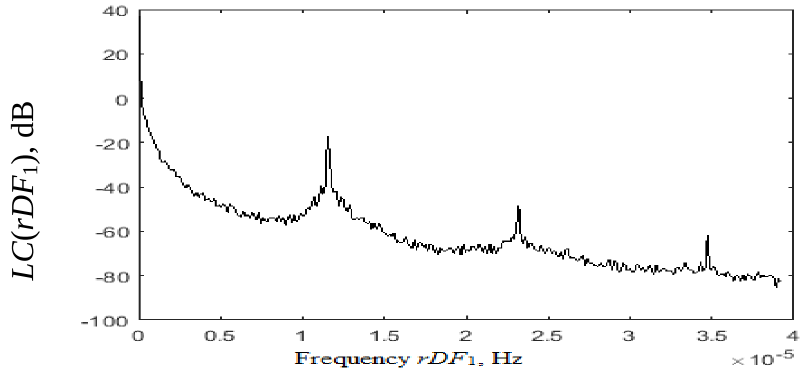
32 days, $n_1 = 50000 \Leftrightarrow 14.08.2013$, $n_1 = 50750 \Leftrightarrow 15.09.2013$

Discrete Fourier transform (DFT): $S(T_0 n)$, $1 \le n \le N_f$

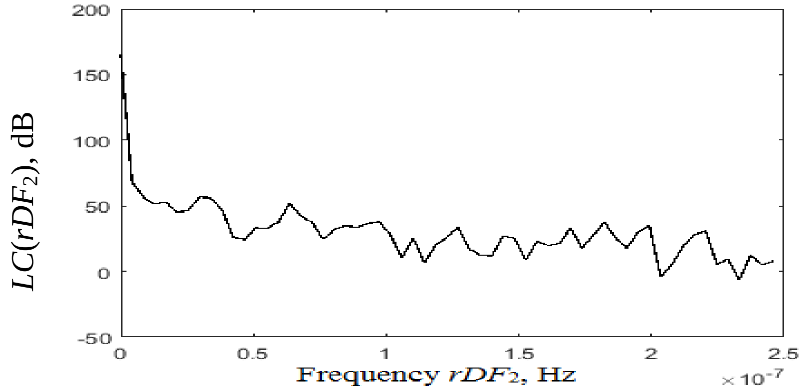
$$N_{1l} \le n \le N_{2l}, \quad N_{1l} = 1 + N(l-1) \quad N_{2l} = N_{1l} + Nl, \quad l = 1,..,l_0$$

$$C_{0l}(r) = \frac{1}{N} \sum_{s=N_{1l}}^{N_{2l}} S(T_0 s) \exp(-jrs/N), \; r = 0,1,...,N-1, \; C_l(r) = C_{0l}^*(r) C_{0l}(r)$$

$$C(r) = \frac{1}{l_0} \sum_{l=1}^{l_0} C_l(r), \quad LC(r\Delta f) = 20 \log_{10}(C(r\Delta f)), \quad \Delta f = 1/NT_0 .$$



Diurnal spectral components, DFT $N = 2048$



Annual spectral components, DFT $N = 16384 * 4$

4.Methods and algorithms for low-pass MH data filtering

4.1.Serial and parallel low-pass filtering for MH matrix data time series

1. 1-minute -sampled MH data time series

Given i, j , $Y(i, j, Tk), k = 1, 2, \ldots$, $\Rightarrow Y_{\Phi}(i, j, Tk) = ?$

2.Serial filtering – noise reduction

Scalar FIR filters

$$Y_{\Phi}(i, j, Tk) = \sum_{s=0}^{s_0} a_s Y(i, j, T(k-s)), \; k = 1, 2, \ldots$$

Cutoff frequency $f_c \Rightarrow ?$, $a^{\circ} \Rightarrow ?$

Periods of components: 11-year solar cycle T_{SN} , annual cycle T_Y , 27-day solar cycle T_{27S} , diurnal Earth cycle T_S

Phase shift correction: $Y_{\Phi}(i, j, Tk) \Rightarrow Y_{\Phi 0}(i, j, Tk)$

$$S_{\Phi}(Tk) = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} Y_{\Phi}(i, j, Tk), \qquad F(S, S_{\Phi}, k_d) = \sum_{k=k_1}^{k_2} (S(Tk) - S_{\Phi}(T(k-k_d))^2,$$

$$k_d^{\circ} = \arg\{ \min_{1 \le k_d \le k_{d0}} F(S, S_{\Phi}, k_d) \}. \qquad Y_{\Phi 0}(i, j, Tk) = Y_{\Phi}(i, j, T(k - k_d^{\circ}))$$

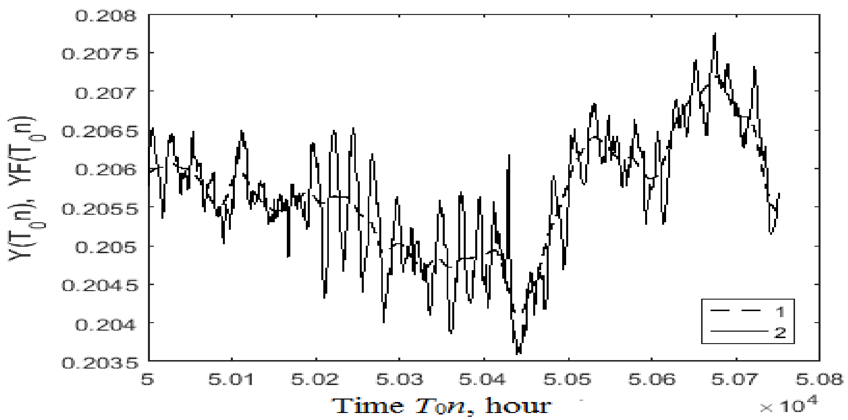
3.Parallel filtering – noise reduction

$Y(Tk), (N_2, N_1)$, matrices of coefficients $A_s, (N_2, N_1)$, $s = 0, 1, \ldots, s_0$, where "$\circ$"denotes elementwise matrix multiplication

Matrix FIR filters

$$Y_{\Phi}(Tk) = \sum_{s=0}^{s_0} A_s \circ Y(T(k-s)), \quad k = 1, 2, \ldots,$$

4.Diurnal variation elimination in 1-hour-sampled MH data, the cutoff frequency $f_c = 0.8 \cdot f_S$, $f_S = 1/T_S$



4.2. One-dimensional low-pass filtering algorithm for MH data based on local sliding models and weighted averaging

1. Given i_0, j_0, data $Y(i_0, j_0, Tk) = Y(Tk)$, $k = 1, 2, \ldots k_f$

High-frequency noise elimination
$Y(i_0, j_0, Tk) \Rightarrow Y_\Phi(i_0, j_0, Tk), k = 1, 2, \ldots k_f$

Sliding local intervals N with an overlap N_d,
$N_{1l} = 1 + N_d(l-1)$, $N_{2l} = N_{1l} + N - 1$, $l = 1, \ldots, l_0$
$l_0 = \arg\{\max_{l>0} l\}$, assuming $N_d(l-1) + N - k_0 < 0$

Local sliding models $Y_M(c_l, Tk)$, $c_l^T = (c_{1l}, c_{2l}, \ldots, c_{ml})$
$Y_M(c_l, Tk) = 0$ for $k < N_{1l}, k > N_{2l}$, $l = 1, \ldots, l_0$

Local functionals $S_{01}(c_l, Y) = \sum_{k=N_{1l}}^{N_{2l}} (Y(Tk) - Y_M(c_l, Tk))^2$,

$c_l^\circ = \arg\{\min_c S_{01}(c_l, Y)\}$,

$E_{0l}(Tk) = 1$ for $N_{1l} \le k \le N_{2l}$, $E_{0l}(Tk) = 0$, for $k < N_{1l}, k > N_{2l}$,

$l = 1, \ldots, l_0$, $E_0(Tk) = \sum_{l=1}^{l_0} E_{0l}(Tk)$,

Weight function $W(Tk) = 1/E_0(Tk)$, $k = 1, 2, \ldots k_f$

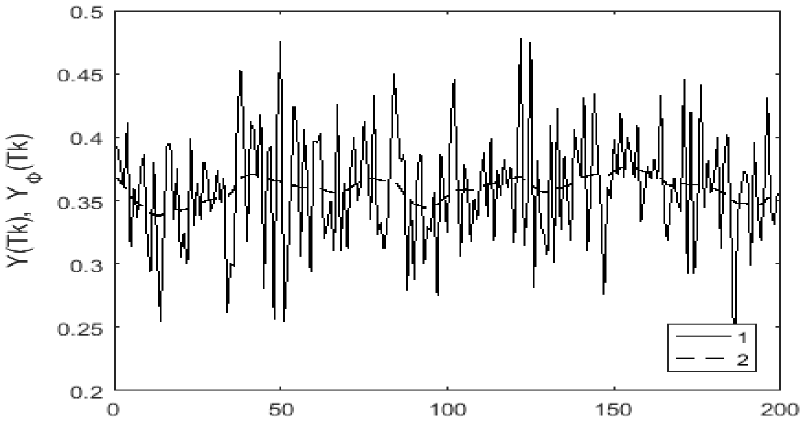
Filtering result

$Y_M(c^\circ, Tk) = W(Tk)\sum_{l=1}^{l_0} Y_M(c_l^\circ, Tk)$, $Y_\Phi(Tk) = Y_M(c^\circ, Tk)$, $k = 1, 2, \ldots k_f$

2. Experimental 1-minute-sampled data $Y(i_0, j_0, Tk) = Y(Tk)$,
$k = 1, 2, \ldots k_f$, $k_f = 500$ $i_0 = 30$, $j_0 = 35$; record time, 15.08.2014, 00.00-08.20 – 8 hours 20 minutes.

Piecewise linear local models $Y_M(c_l, Tk) = c_{0l} + c_{1l}Tk$,
$T = 60c$, $N = 10, N_d = 2$

One-dimensional filtering result $Y_\Phi(Tk)$, $k = 1, 2, \ldots k_f$



4.3. Two-dimensional low-pass filtering algorithm for MH data based on local sliding models and weighted averaging

1. Given k_0, data matrix $Y(i,j,Tk_0)=Y(i,j)$, $i=1,\ldots,N_1$, $j=1,\ldots,N_2$

Two-dimensional high-frequency noise elimination

$Y(i,j)\Rightarrow Y_\Phi(i,j)$

Two-dimensional sliding local intervals with an overlap $\Delta N_1, N_{d1}$, $\Delta N_2, N_{d2}$

$N_{1k}\le i\le N_{2k}$, $N_{1k}=N_{d1}(k-1)$, $N_{2k}=N_{1k}+\Delta N_1-1$, $k=1,\ldots,m_{01}$,

$N_{1n}\le j\le N_{2n}$, $N_{1n}=N_{d2}(n-1)$, $N_{2n}=N_{1n}+\Delta N_2-1$, $n=1,\ldots,m_{02}$.

Local sliding models $Y_M(c_{kn},i,j)$, $N_{1k}\le i\le N_{2k}$, $N_{1n}\le j\le N_{2n}$, $Y_M(c_{kn},i,j)=0$, $i<N_{1k}, N_{2k}<i$, $j<N_{1n}, N_{2n}<j$

Two-dimensional local functional

$$S_{02}(Y,c_{kn})=\sum_{i=N_{1k}}^{N_{2k}}\sum_{j=N_{1n}}^{N_{2n}}(Y(i,j)-Y_M(c_{kn},i,j))^2$$

$$c^{\circ}_{kn}=\arg\{\min_{c_{kn}} S_{02}(Y,c_{kn})\}$$

Weight function

$E_{0kn}(i,j)=1$ for $N_{1k}\le i\le N_{2k}$, $N_{1n}\le j\le N_{2n}$,

$E_{0kn}(i,j)=0$ for $i<N_{1k}, N_{2k}<i$, $j<N_{1n}, N_{2n}<j$

$$E_0(i,j)=\sum_{k=1}^{m_{01}}\sum_{n=1}^{m_{02}}E_{0kn}(i,j),\quad W(i,j)=1/E_0(i,j)$$

Two-dimensional filtering result

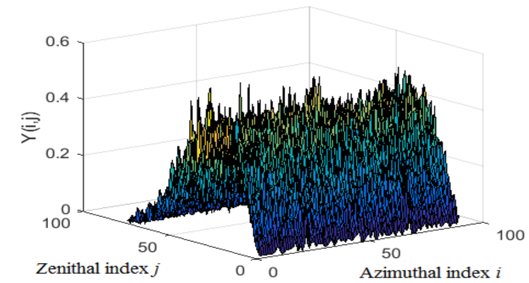
$$Y_M(c^{\circ},i,j)=W(i,j)\sum_{k=1}^{m_{01}}\sum_{n=1}^{m_{02}}Y_M(c^{\circ}_{kn},i,j),\quad Y_\Phi(i,j)=Y_M(c^{\circ},i,j)$$

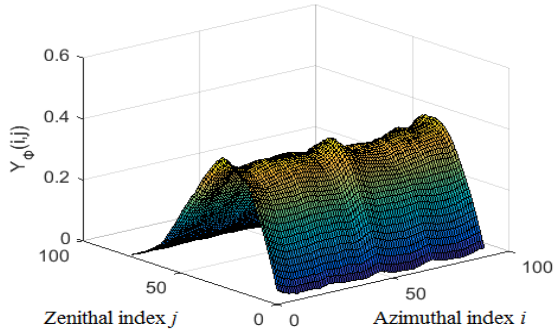
2. Experimental 1-minute-sampled matrix

$Y(i,j,Tk_0)$, 15.08.2018, $k_0=12$

Piecewise linear local models $Y_M(c_{kn},i,j)=c_{0,kn}+c_{1,kn}i+c_{2,kn}j$

Two-dimensional (spatial) filtering result $Y_\Phi(i,j)$





5. A method for normalized HF calculation and variations -1- for muon flux intensity distribution functions (MFIDF), recognition of EEH – local anisotropy (LA) for MH data

1. 1-minute-sampled matrix MH data -output MFIDF $Y(i, j, Tk)$, $k = 1,\dots,k_{f1}$, $i = 1,\dots,N_1, j = 1,\dots,N_2$.

Hardware function (HF) parametric model - $a_{0,ij}$, $i = 1,\dots,N_1, j = 1,\dots,N_2$.

Input MFIDF math. expectations $Y_0(i, j, Tk)$, $k = 1,2,\dots$ - c_0.

Functional $S_0(a,Y) = \sum_{k=1}^{k_{f1}} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (Y(i, j, Tk) - a_{ij})^2$, $a_{ij} = a_{0,ij} c_0$,

Normalized HF $A_N^{\circ}(i, j) = a_{ij}^{\circ} = \frac{1}{k_f} \sum_{k=1}^{k_{f1}} Y(i, j, Tk)$, $i = 1,\dots,N_1, j = 1,\dots,N_2$.

Input MFIDF variation-1 estimates $\delta Y^{\circ}(i, j, Tk) = Y(i, j, Tk) - A_N^{\circ}(i, j)$, for $k = k_{f1}+1,\dots,k_{f1}+k_{f2}$

Input MFIDF normalized variation-1 estimates

$$\delta Y_N^{\circ}(i, j, Tk) = (Y(i, j, Tk) - A_N^{\circ}(i, j)) / A_N^{\circ}(i, j),\ k = k_{f1}+1,\dots,k_{f1}+k_{f2}$$

Averaged estimate $\delta \bar{Y}_N^{\circ}(i, j)$

$$\delta \bar{Y}_N^{\circ}(i, j) = \frac{1}{k_{f2}} \sum_{k=k_{f1}+1}^{k_{f1}+k_{f2}} \delta Y_N^{\circ}(i, j, Tk).$$

Two-dimensional filtering with local models

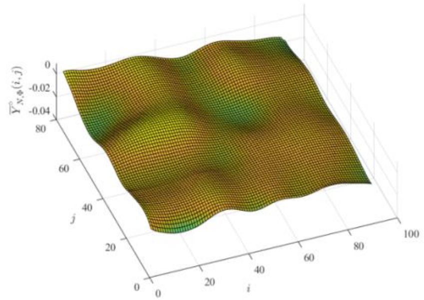
$\delta \bar{Y}_N^{\circ}(i, j) \Rightarrow \delta \bar{Y}_{N,\Phi}^{\circ}(i, j)$

2. Experimental 1-minute-sampled MH data $Y(i, j, Tk)$, $k = k_{f1}+1,\dots,k_{f1}+k_{f2}$, 15.08.2014

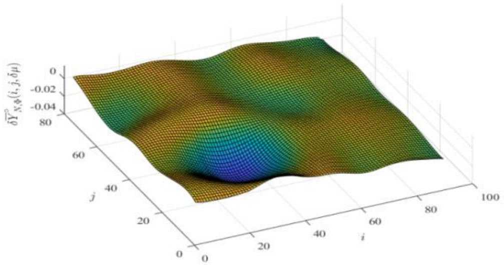
Model Forbush decreases $\mu(i, j)$, $i = 1,\dots,N_1$, $j = 1,\dots,N_2$,

$\mu(i, j) = 1 - \delta\mu$ for $i_1 \le i \le i_2$, $j_1 \le j \le j_2$, otherwise - $\mu(i, j) = 1$

$Y(i, j, Tk, \mu) = Y(i, j, Tk)\ \mu(i, j)$



Estimate of the average filtered normalized variation-1 of the input MFIDF $\delta \bar{Y}_{N,\Phi}^{\circ}(i, j)$, $\delta\mu = 0$



Estimate of the average normalized variation-1 of the input MFIDF $\delta \bar{Y}_{N,\Phi}^{\circ}(i, j, \delta\mu)$, $\delta\mu = 0.03$ - EEH – LA recognition

6. A method for calculation of normalized MFIDF variations – 2, recognition of EEH for MH data

1. 1-hour-sampled normalized Poisson-distributed MH data
$Y(i,j,T_0 n)$, $n = n_0,...,n_f$, $i = 1,...,N_1$, $j = 1,..,N_2$, i, j, $N_1 \cdot N_2$

High-frequency noise elimination

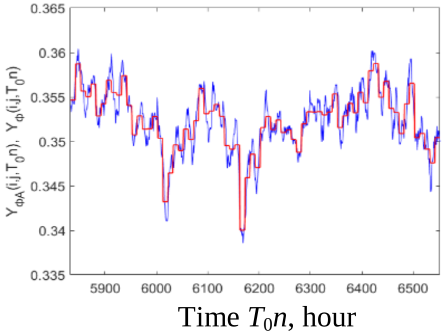
r_0, w_c, $Y(i,j,T_0 n) \Rightarrow Y_\Phi(i,j,T_0 n)$, $n = n_0,...,n_f$

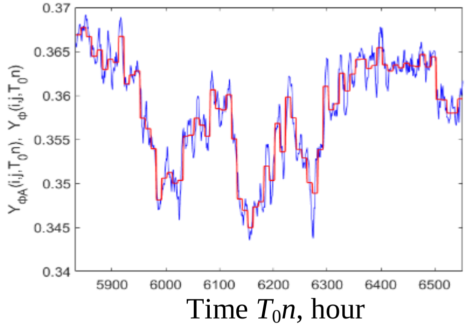
Approximation $Y_\Phi(i,j,T_0 n)$ using local piecewise constant models, local intervals

Δn, $n_l = n_0 + \Delta n(l-1)$, $n_{l+1} = n_l + \Delta n - 1$, $l = 1,...,l_0$ i, j, $Y_\Phi(i,j,T_0 n) \Rightarrow$
$Y_{\Phi A}(i,j,T_0 n)$

$Y_{\Phi A,l}(i,j) = \dfrac{1}{\Delta n}\sum_{n=n_l}^{n_{l+1}} Y_\Phi(i,j,T_0 n)$, $Y_{\Phi A}(i,j,T_0 n) = Y_{\Phi A,l}(i,j)$, for $n_l \le n < n_{l+1}$,

$Y_{\Phi A}(i,j,T_0 n) = 0$, $n < n_l, n_{l+1} < n$, $l = 1,...,l_0$, $N_1 \cdot N_2$





Data interval 01.09.2017-30.09.2017 (720 hours), $n_0 = 5832$
$n_f = 6552$, $\Delta n = 10$; $i = 30, j = 20$, $b, i = 30, j = 25$

Calculation of functions of normalized variations $\delta Y(i,j,T_0 n)$ for filtered MH data

$$Y_{\Phi 0}(i,j) = \frac{1}{n_f}\sum_{n=n_0}^{n_f} Y_\Phi(i,j,T_0 n), \quad \delta Y_A(i,j,T_0 n) = \frac{Y_{\Phi A}(i,j,T_0 n) - Y_{\Phi 0}(i,j)}{Y_{\Phi 0}(i,j)}$$

2. Temporal filtering - approximation using local piecewise constant models $\delta Y(i,j,T_0 n) \Rightarrow \delta Y_A(i,j,T_0 n)$, $n = n_0,...,n_f$

$\delta Y_{A,l}(i,j) = \dfrac{1}{\Delta n}\sum_{n=n_l}^{n_{l+1}} \delta Y(i,j,T_0 n)$, $\delta Y_A(i,j,T_0 n) = \delta Y_{A,l}(i,j)$, for $n_l \le n < n_{l+1}$,
$l = 1,...,l_0$

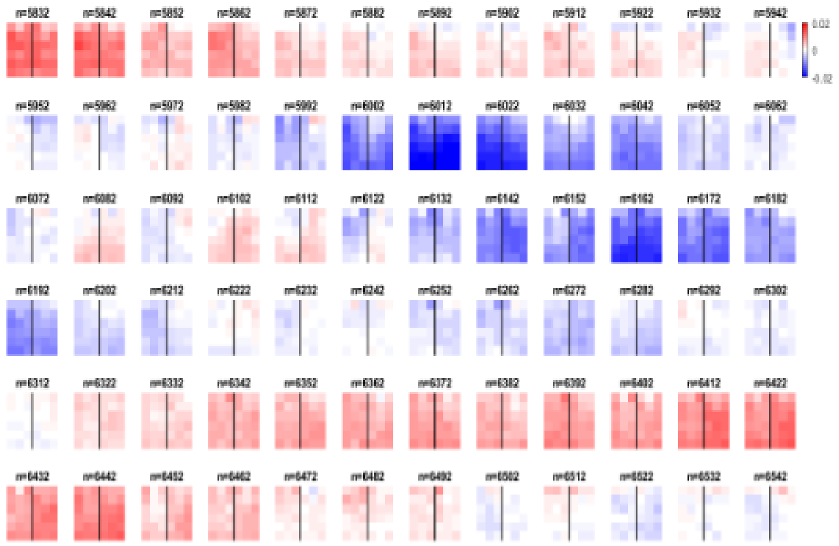
6. A method for calculation of normalized MFIDF variations – 2, recognition of EEH for MH data

3. Two-dimensional filtering of normalized variation functions local approximation rectangles $\Delta N_1, \Delta N_2$, $\bar{p}\Delta N_1 = \bar{N}_1 \le N_1$

$\bar{q}\Delta N_2 = \bar{N}_2 \le N_2$, $\bar{p}, \bar{q}$, $p,q \Leftrightarrow i,j$,

$1+(p-1)\Delta N_1 \le i \le p\Delta N_1$, $p=1,..,\bar{p}$, $1+(q-1)\Delta N_2 \le j \le q\Delta N_2$, $q=1,..,\bar{q}$.

$$\delta Y_A(p,q,T_0 n) = \frac{1}{\bar{p}\bar{q}} \sum_{i=1+(p-1)\Delta N_1}^{p\Delta N_1} \sum_{j=1+(q-1)\Delta N_2}^{q\Delta N_2} \delta Y(i,j,T_0 n),\ p=1,..,\bar{p},\ q=1,..,\bar{q}$$



$n_0 = 5832$ $n_f = 6552$, $\Delta n = 10$.

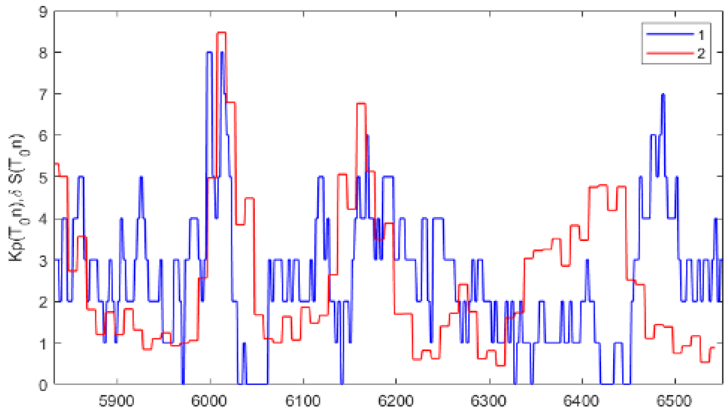
Recognition of EEH areas with LA for $\delta Y_A(p,q,T_0 n)$; LA (5982-6042), (6132-6182), (6412-6442)

$\Delta N_1 = 10, \Delta N_2 = 10$, $\bar{p} = 9$, $\bar{q} = 7$, subarea №1- $p = 2 \div 4, q = 2 \div 6$, subarea №2- $p = 6 \div 8, q = 2 \div 6$; №1: $i = 37^\circ \div 144^\circ, j = 11^\circ \div 60^\circ$,

№2: $i = 217^\circ \div 324^\circ, j = 11^\circ \div 60^\circ$

4. Geomagnetic activity index function $K_p(T_0 n)$, function of average intensity of normalized variations $\delta S(T_0 n)$,

$$\delta S^2(T_0 n) = \frac{\alpha_C}{\bar{p}\bar{q}} \sum_{p=1}^{\bar{p}} \sum_{q=1}^{\bar{q}} \delta Y^2(p,q,T_0 n),\ \alpha_C,\ n = n_0,...,n_f$$



7. Method for calculating indicator functions, recognition of EEH (LA) for MH data

1. 1-minute-sampled normalized Poisson-distributed matrix MH data

$Y(i,j,Tk)$, $k=1,2,\dots$, $i=1,\dots,N_1$, $j=1,\dots,N_2$, $\xi_k(i,j)=Y(i,j,Tk)$

LA areas $\Psi_{a,k} \Rightarrow ?$ $(i,j)\in\Psi_{a,k}\subset\Psi_0$, $k=1,2,\dots$ $\Psi_0=\{(i,j): i=1,\dots,N_1, j=1,\dots,N_2\}$

Mathematical expectations $\lambda^\circ = \lambda^\circ(i,j)=\frac{1}{k_f}\sum_{k=1}^{k_f}\xi_k(i,j)$, confidence level - γ,

$\Phi(\delta_\gamma)=\gamma/2$, confidence intervals - $h_{mn}\le\bar{\lambda}\le h_{mx}$,

$$h_{mn}=\{-\delta_\gamma/(2\sqrt{k_f})+\sqrt{\lambda^\circ-\delta_\gamma^2/4k_f}\}^2, h_{mx}=\{\delta_\gamma/(2\sqrt{k_f})+\sqrt{\lambda^\circ+\delta_\gamma^2/4k_f}\}^2,$$

Reference and current time sections $n=1,\dots,n_0+n_1$

$k_{0,1}\le k\le k_{0,2}$, $k_{0,1}=1, k_{0,2}\le k_f$ reference section,

$k_{n,1}\le k\le k_{n,2}$, where $k_{n,1}=k_f+1+(n-1)k_f$, $k_{n,2}=k_f+1+nk_f$, $n=1,\dots,n_0+n_1$,

Reference and current confidence intervals $(h_{e,mn},h_{e,mx})\Leftrightarrow(h_{n,mn},h_{n,mx})$

Anomalousness indicators $a_n=a(h_{e,mn},h_{e,mx},h_{n,mn},h_{n,mx})$, $a_n(i,j)$, $dh_n=h_{n,mx}-h_{n,mn}$

1.if $(h_{n,mx}\le h_{e,mn})$, then $a_n=(h_{e,mn}-h_{n,mn})/dh_n$,

2.if $(h_{n,mn}\le h_{e,mn})$ and $(h_{n,mx}>h_{e,mn})$ and $(h_{n,mx}<h_{e,mx})$, then $a_n=(h_{e,mn}-h_{n,mn})/dh_n$,

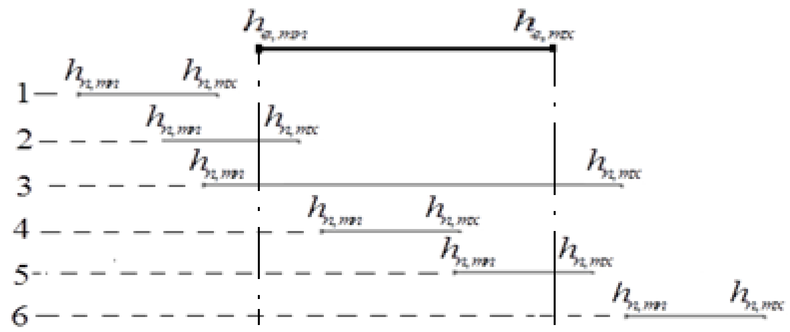
3.if $(h_{n,mn}\le h_{e,mn})$ and $(h_{n,mx}>h_{e,mx})$, then $a_n=((h_{e,mn}-h_{n,mn})+(h_{n,mx}-h_{e,mx}))/dh_n$,

4.if $(h_{n,mn}>h_{e,mn})$ and $(h_{n,mx}<h_{e,mx})$, and $a_n=0$,

5.if $(h_{n,mn}>h_{e,mn})$ and $(h_{n,mn}<h_{e,mx})$ and $(h_{n,mx}>h_{e,mx})$, then

$a_n=(h_{n,mx}-h_{e,mx})/dh_n$,

6.if $(h_{n,mn}>h_{e,mx})$, then $a_n=(h_{n,mx}-h_{e,mx})/dh_n$.



Anomalousness matrices $a_n(i,j)\Leftrightarrow A_n$,

Threshold matrix $a_0(i,j)\Leftrightarrow A_0$, $n=1,\dots,n_0$ $a_0(i,j)=\frac{1}{n_0}\sum_{n=1}^{n_0}a_n(i,j)$

Indicator matrices G_n, $n=n_0+1,\dots,n_0+n_1$. $g_n(i,j)$,

$i=1,\dots,N_1$, $j=1,\dots,N_2$, $g_n(i,j)=1$, if $a_n(i,j)\ge a_0(i,j)$, $g_n(i,j)=0$, if $a_n(i,j)<a_0(i,j)$.

7. Method for calculating indicator functions, recognition of EEH (LA) for MH data

2. Temporal filtering $G_{\Phi 0}(n_1)$, $n = n_0+1,\dots,n_0+n_1$

$g_{\Phi 0}(n_1,i,j)$, $\bar{g}_{\Phi 0}(n_1,i,j) = \frac{1}{n_1}\sum_{n=1}^{n_1} g_n(i,j)$, threshold $\bar{\bar{g}}_{\Phi 0}(n_1)$

$g_{\Phi 0}(n_1,i,j)=1$, if $\bar{g}_{\Phi 0}(n_1,i,j) \ge \bar{\bar{g}}_{\Phi 0}(n_1)$, $g_{\Phi 0}(n_1,i,j)=0$, if $\bar{g}_{\Phi 0}(n_1,i,j) < \bar{\bar{g}}_{\Phi 0}(n_1)$,

3. Spatial filtering $G_{\Phi 1}(n_1)$

$\Delta N_1, \Delta N_2$, $\bar{p}\Delta N_1 = \bar{N}_1 \le N_1$, $\bar{q}\Delta N_2 = \bar{N}_2 \le N_2$, $\bar{p},\bar{q}$

$1+(p-1)\Delta N_1 \le i \le p\Delta N_1$, $p=1,..,\bar{p}$, $1+(q-1)\Delta N_2 \le j \le q\Delta N_2$, $q=1,..,\bar{q}$

$\bar{g}_{\Phi 1}(n_1,p,q) = \sum_{i=1+(p-1)\Delta N_1}^{p\Delta N_1} \sum_{j=1+(q-1)\Delta N_2}^{q\Delta N_2} g_{\Phi 0}(n_1,i,j)$, $p=1,..,\bar{p}$, $q=1,..,\bar{q}$, threshold $\bar{\bar{g}}_{\Phi 1}(n_1)$

$g_{\Phi 1}(n_1,p,q)=1$, if $\bar{g}_{\Phi 1}(n_1,p,q) \ge \bar{\bar{g}}_{\Phi 1}(n_1)$, $g_{\Phi 1}(n_1,p,q)=0$, if $\bar{g}_{\Phi 1}(n_1,p,q) < \bar{\bar{g}}_{\Phi 1}(n_1)$

4. Experimental MH data and model decreases

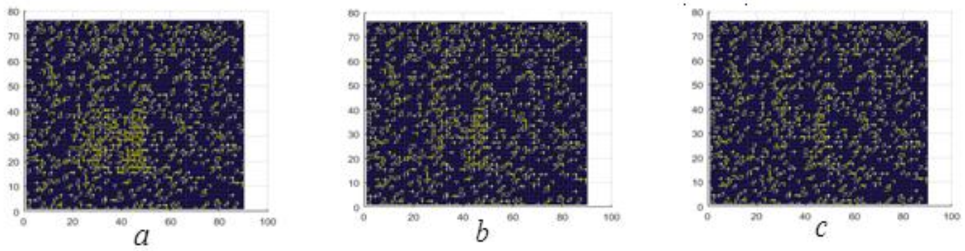
$Y_E(i,j,Tk)$, $k=1,2,\dots$; $\mu(i,j) = 1-\delta\mu$, for $i_1 \le i \le i_2$, $j_1 \le j \le j_2$,

$\mu(i,j)=1$ for $i_1 < i, i_2 > i$, $j_1 < j, j_2 > j$.

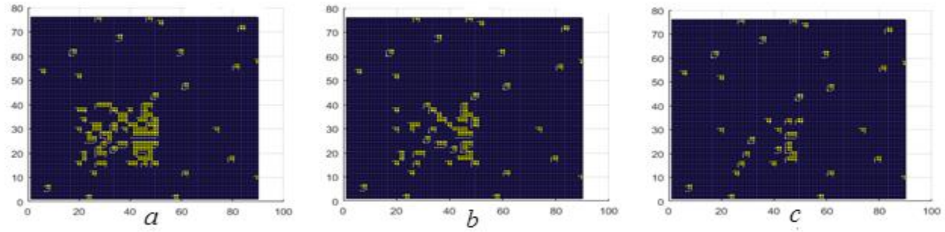
$Y_E(i,j,Tk,\mu) = Y_E(i,j,Tk)\mu(i,j)$, $k = k_f(n_0+1)+1,\dots,k_f(n_0+n_1+1)$.

5. An example of EEH (LA) recognition

Temporal filtering $G_{\Phi 0}(n_1)$, $a - G_{\Phi 0}(n_1,\delta\mu_1)$, $\delta\mu_1 = 0.05$; $b - G_{\Phi 0}(n_1,\delta\mu_2)$, $\delta\mu_2 = 0.04$; $c - G_{\Phi 0}(n_1,\delta\mu_3)$, $\delta\mu_3 = 0.03$.



Spatial filtering $G_{\Phi 1}(n_1,\delta\mu)$, $\Delta N_1 = \Delta N_2 = 4$, $a - G_{\Phi 1}(n_1,\delta\mu_1)$, $\delta\mu_1 = 0.05$; $b - G_{\Phi 1}(n_1,\delta\mu_2)$, $\delta\mu_2 = 0.04$; $c - G_{\Phi 1}(n_1,\delta\mu_3)$, $\delta\mu_3 = 0.03$.



Thank you for attention !