Mean depth of shower and interaction characteristics

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Shower & hadronic interactions

L. Kheyn, Shower center of gravity and hadronic interaction characteristics, Astropart. Phys. **92**, 7 (2017).

Which hadron-air interaction characteristics define shower longitudinal development?

- ❑ **Cross-sections?**
- ❑ **Inelasticities?**
- ❑ **Multilicities?**
- ❑ **Diffraction?**
- ❑ **Leading effect**

Shower & hadronic interactions

The above questions are usually answered with use of Monte Carlo.

But it is very desirable to explicitly connect air shower longitudinal profile, in particular depth of shower maximum (X_{max}), with hadronic interaction properties.

Very approximate approaches have been tried for that, from toy models to extension of Heitler model, developed for electromagnetic shower, to hadronic shower by Matthews. The latter being helpful for study of muon production is not of much use for connecting interaction properties to depth of shower maximum.

Why not to use directly cascade theory?

Because shower maximum is inconvenient quantity for treatment by cascade equations.

Rather, convenient quantity is shower mean depth (center of gravity).

$$\overline{X(E)} = \int_0^\infty X\, N(X)\, dX \Big/ \int_0^\infty N(X)\, dX$$

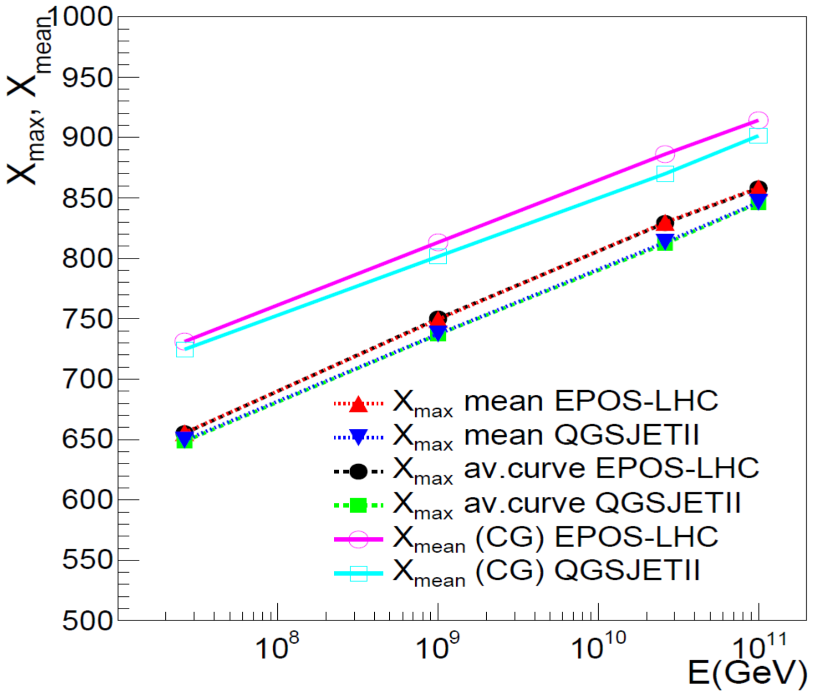
<X> vs X_{max}

Measured quantity is depth of shower maximum.

If mean shower depth (shower center of gravity) is valuable for:

✓estimation of rate of rise of shower maximum with energy

✓difference between generators?



✓Very small difference in slope in <X_{max}>and <X>

✓Same difference between generators in <X_{max}>and <X>

Mean shower depth is suitable for study of shower maximum

Shower & hadronic interactions

Crucial importance of i) cross-section (interaction length), ii) inelasticity, in particular, fraction of hadron energy transferring to electromagnetic component, for shower longitudinal development is out of question.
But why total multiplicity matters?
Because longitudinal shower development is strongly influenced by energy dissipation. This appears due to logarithmic dependence of electromagnetic shower depth on energy. Be that depth independent on energy, energy dissipation in interaction would practically not influence shower longitudinal development.

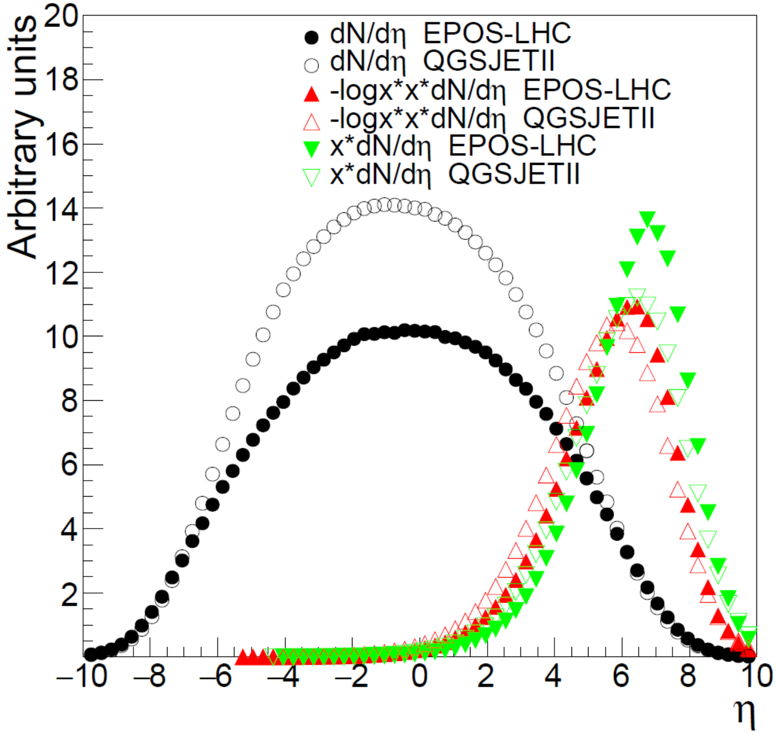
If total multiplicity, that is, integral $\int \frac{dn}{dx}$ dx is a good representative of energy dissipation? Not quite.
Secondary particles do not contribute to lengthening of parent shower with equal weight, their contribution is proportional to their energy and the length of subshower they produce, that is to logarithm of their energy.

Therefore another integral, $\int x \log x \frac{dn}{dx} dx$, should represent energy dissipation.

η distributions for three integrals

Does integral $\int x \log x \frac{dn}{dx} dx$ instead of $\int \frac{dn}{dx} dx$ matter?

pN interactions at 7 TeV



- Integral for multiplicity is accumulated in the central region.
- Integral for energy transfer is accumulated in the forward region.
- Integral for <logx> is accumulated also in the forward region, although in somewhat "less forward" than for energy transfer.
- Difference between generators for that integral is much smaller than for multiplicity integral.

Contribution from first interaction.

Let's check applicability of cascade theory for calculating mean shower depth for the simplest case: electromagnetic contribution from the first interaction of proton.

It is possible to derive exact expression without any simplifying assumptions.

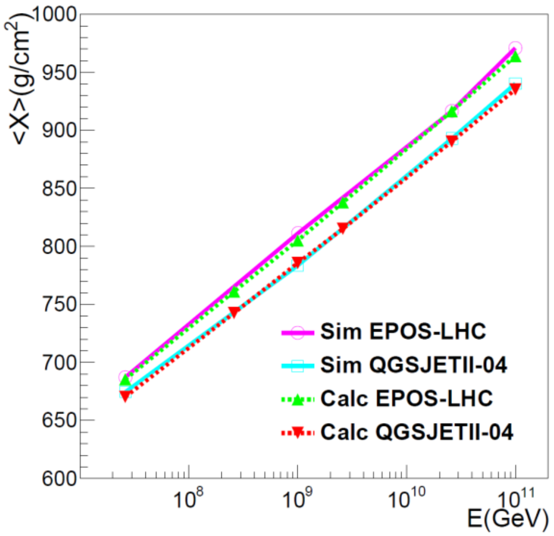
$$<X^{(1)}_{p\gamma}(E)> = \lambda_P + X_0(\log E + 0.7 + g_{p\gamma}/r_{p\gamma})$$

$$g_{p\gamma} = \int x \frac{dn_{p\to\gamma}}{dx} dx$$

$$r_{p\gamma} = \int x \log x \frac{dn_{p\to\gamma}}{dx} dx$$

X_0 is radiation length in air.

If calculations fit simulations?



Exact agreement

Only two kinds of integrals matter.

- One with mean fraction of energy transferred to another component (electromagnetic in that case)
- and second with the same integral but with the weight log(x) added.

Total multiplicity of produced γ does not enter resulting formula, corresponding integral $\int \frac{dN}{dx} dx$ is replaced by integral $\int x \log x \frac{dN}{dx} dx$.

Whole shower

For obtaining expression for the mean shower depth of whole shower several simplifying assumptions are needed:

- Feinman scaling
- Cascading only two types of hadrons: barions (nucleons) and pions
- Neglecting decay of charged pions
- Isotopic symmetry, i.e. similar production of pions of all three charges.
- Logarithmically rising p-air and π-air cross-sections

Within these assumptions, exact solution of equations of cascade theory is possible.

The obtained expression for primary proton of energy E splits into mean depth of purely electromagnetic shower at the energy of primary proton, $<\bar{X}_\gamma>(E)$, and modification of that due to hadronic cascading, $<X>(E) = <X_\gamma>(E) + \Delta X_h$

Moreover, ΔX_h, in turn, splits into contributions of nucleon and pion: $\Delta X_h = \Delta X_N + \Delta X_\pi$,

which have a simple form $\Delta X_i = C_i(\lambda_i + X_0 <\log x>_i)$, where i stands for N and π.

λ_i are interaction lengths taken at some effective, lower than primary, energy,

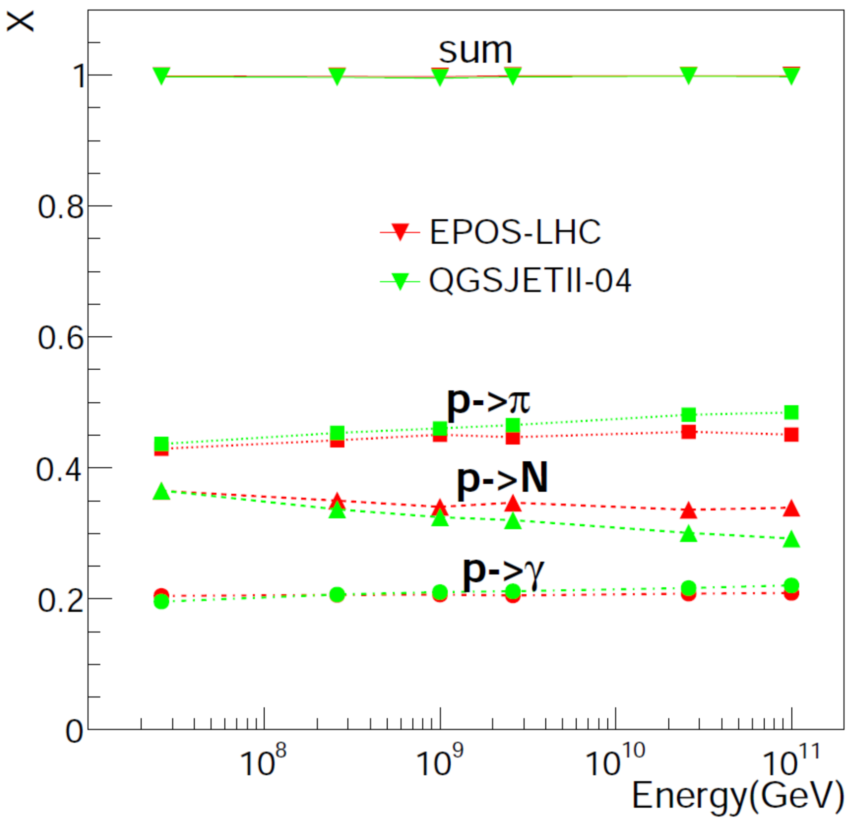
and $<\log x> = \int x \log x \frac{dn}{dx} dx$, where integration proceeds over all produced particles, i.e. average over inclusive spectrum $\frac{dn}{dx}$ done with weight $x*\log x$.

Factors C_i for nucleon and pion are: $C_N = 1/g_{N\pi}$, $C_\pi = (2/3)/g_{\pi\gamma}$.

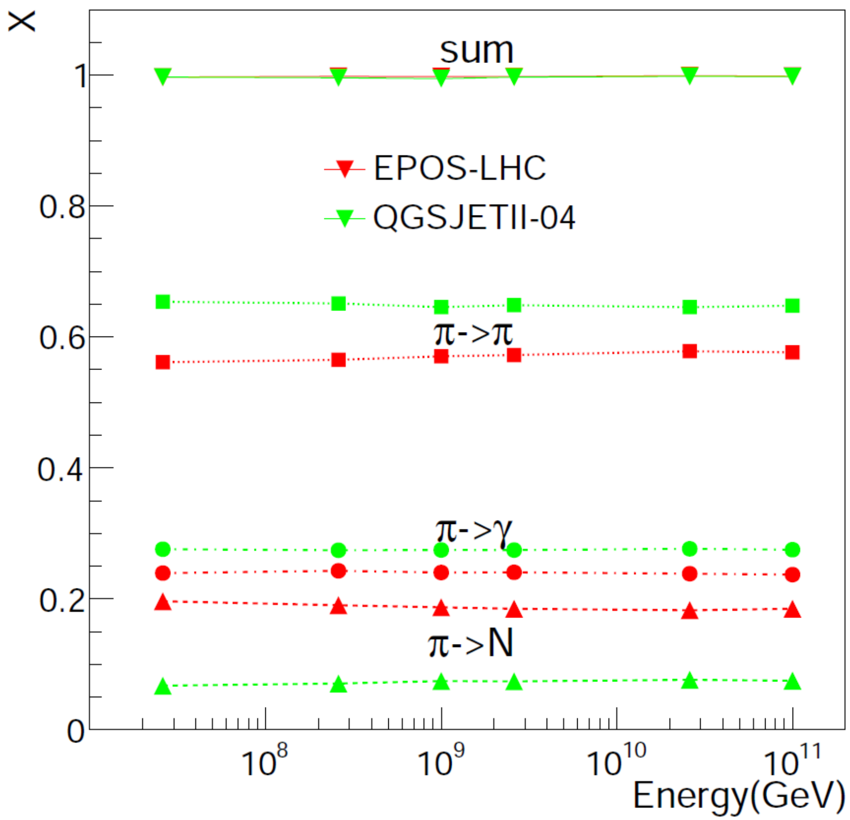
Here $g_{ij} = \int x \frac{dn_{i->j}}{dx} dx$ are energy transfers (e.g. inelasticity), where i and j denote primary and secondary particles.

Energy transfers

Proton



Pion

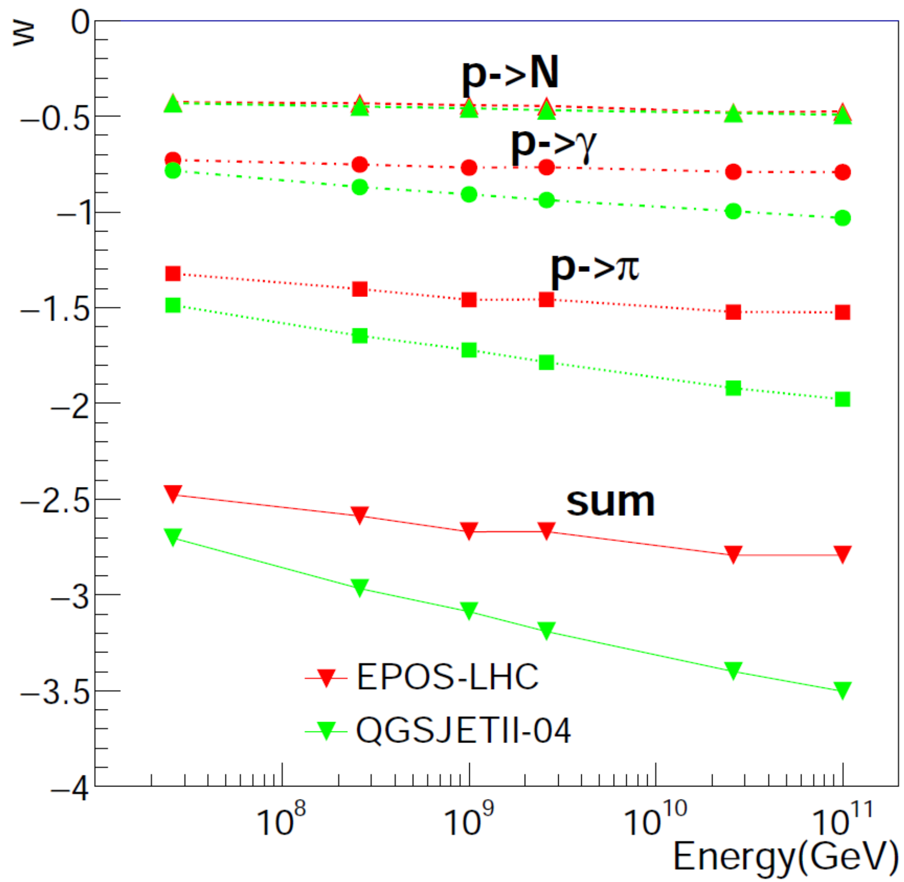


Most crucial energy transfer, p->γ, is energy independent and about same in two generators. Same for p->h transfer.

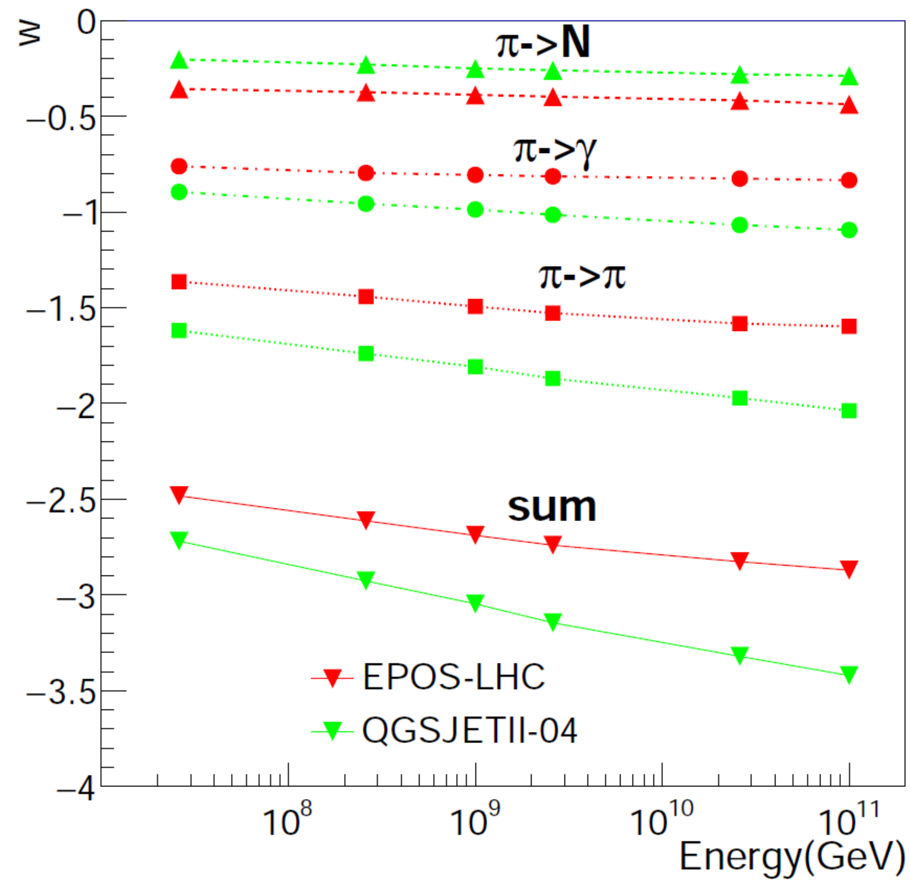
- All energy transfers are energy independent.
- For π->γ, difference between generators is substantial, 16%. That is important!

<logx>

Proton



Pion

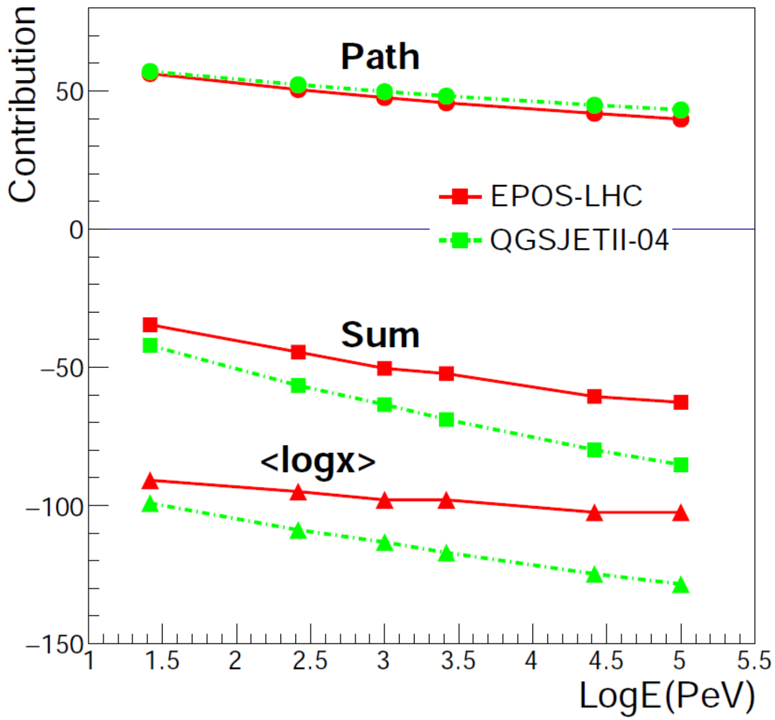


- Clear energy dependence, similar for proton and pion.
- Noticable difference between generators, both in absolute value and in energy dependence, where QGSJETII produces larger multiplicity and stronger energy dependence.

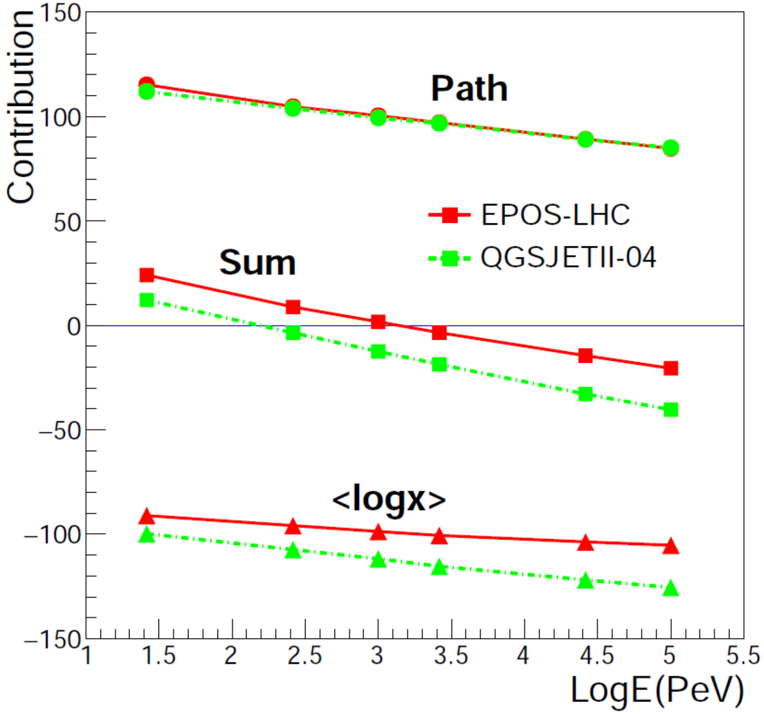
Contributions from λ and <logx>

Contributions of path, $C_i \lambda_i$, and <logx> , $C_i X_0\langle \log x\rangle_i$ in hadronic modification terms.

Proton

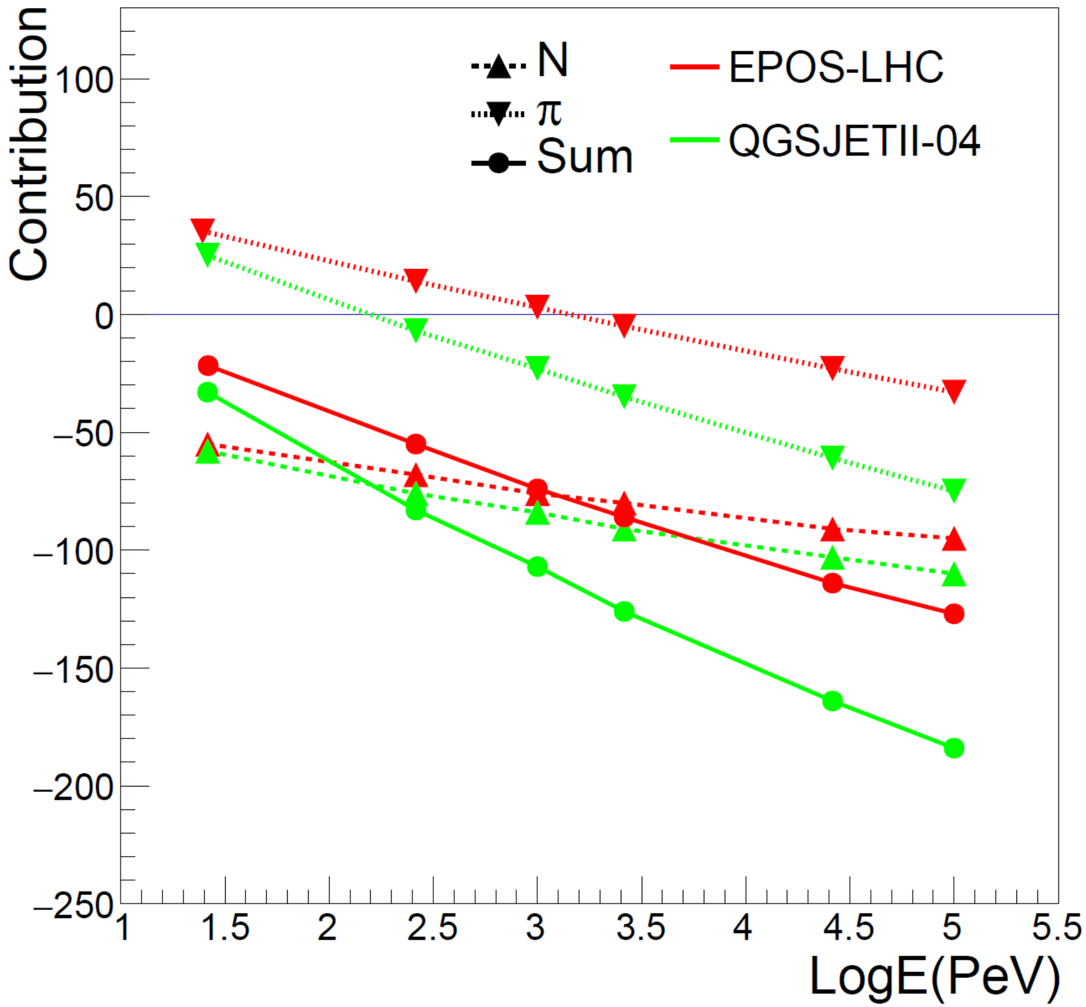


Pion



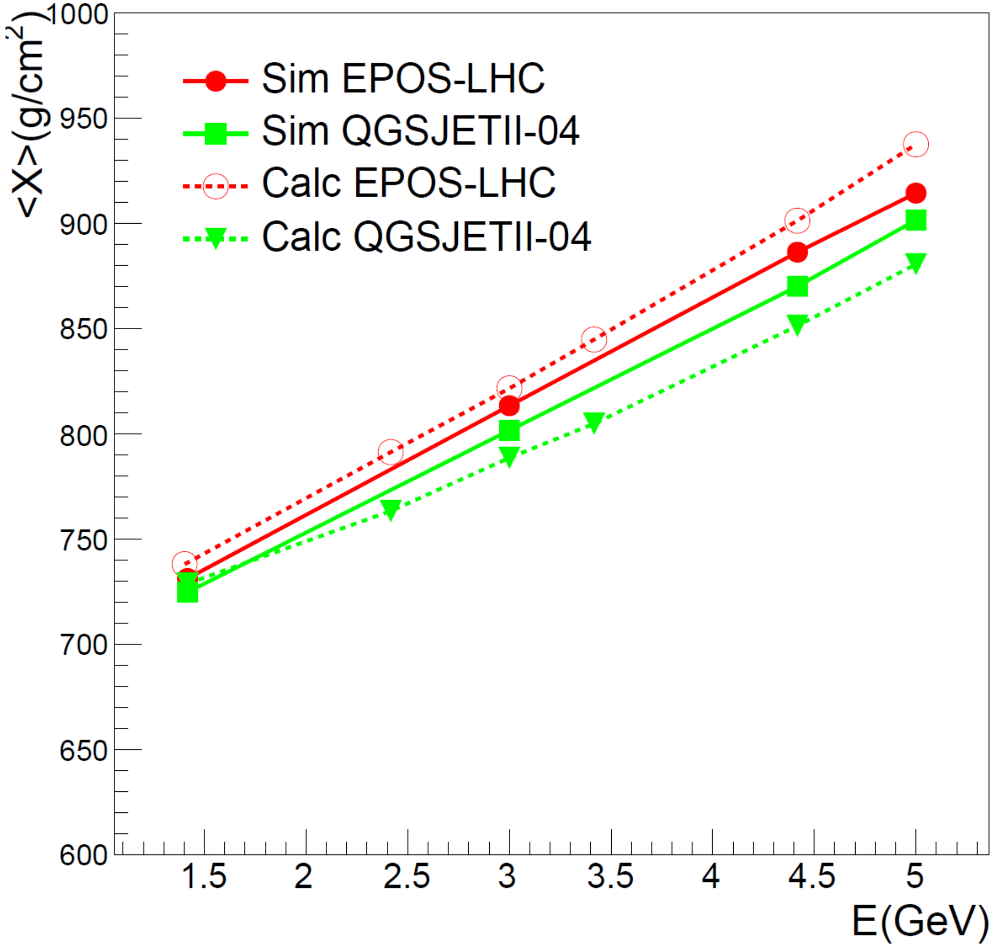
- Contributions of path are practically same for two generators.
- The difference between generators comes from <logx> terms.
- For proton, negative contribution of <logx> overweights positive contribution of path.
- For pion, that holds at high energies, but at smallest energies, path overweights <logx>.

Contributions of nucleon and pion



Main difference between generators comes from nucleon contribution.

Comparison with simulation



Qualitatively, calculations provide similar results to simulations.
Quantitatively, calculations provide larger difference between generators than simulations.

Discussion

Main limitation to the approach comes from the accounting for scaling violation. It is accounted to first order since inclusive spectra for each primary proton energy are taken from Monte Carlo at corresponding energy, that is first interaction is described without simplifications.
But for following cascade generations, which happen at lower energies, the approach uses same inclusive spectra as for the first interaction which thus leads to overestimation of multiplicity and accordingly to larger absolute (which is negative) value of hadronic contribution. Difference between generators corresponds to initial energy and thus is exaggerated since at lower energies sequential interactions it should be smaller.

Conclusions

Cascade theory provides opportunity to obtain, within some simplifying assumptions, expression for mean depth of shower produced by proton in air. That expression proves to be very convenient for analysis of influence of hadronic interaction characteristics on longitudinal shower development.
The expression splits into contributions of electromagnetic shower and its hadronic modification, which in turn splits into contributions of nucleon and pion.
New result is that energy dissipation in course of particle multiplication is characterized not, as it conventially considered, by mean multiplicity of produced in interaction particles, $\int \frac{dn}{dx} dx$, but “forward multiplicity”, $\int x \log x \frac{dn}{dx} dx$.
This player in shower life proves to be the main one defining difference between generators both in absolute values of mean shower depth and in its energy dependence.