Is Lorentz Invariance Violation the Key to the Muon Puzzle?

(Based on arXiv:2412.08349)

N. Martynenko[1,2] G. Rubtsov[1] P. Satunin[1]
A. Sharofeev[1,2] S. Troitsky[1,2]

[1]INR RAS
[2]MSU

ISCRA-2025
June 25, 2025

How to construct Lorentz-invariance violation theories?

1. Kinematical approach:

$$E^2 = k^2 + \sum_{n=1,2,\ldots} s_n \frac{k^{2+n}}{M^n_{\mathrm{LIV},(n)}}, \tag{1}$$

   Violation can be different: «subluminal» ($s_n = -1$) и «superluminal» ($s_n = +1$). Different regimes can lead to different physical consequences.

2. Effective field theory approace:

$$\mathcal{L}_{\mathrm{LIV}} \supset \mathcal{L}_{\mathrm{SM}} + \sum_n \frac{1}{\Lambda_n} \mathcal{O}_n. \tag{2}$$

What does Lorentz invariance violation lead to. . .

1. Shifted reaction thresholds, other decay widths, scattering cross-sections, time delays in propagation over large distances (new phase velocities), birefringence... — a large field for phenomenology.
2. New processes prohibited by conservation laws in LI physics.

Current constraints

e^-/γ	Test of QG	Sub(-) or super(+) luminal	Limits			Source	Ref.
			$\lvert\xi_0\rvert(\lvert\eta_0\rvert)$	$E_{\rm LIV}^{(1)}$ (eV)	$E_{\rm LIV}^{(2)}$ (eV)		
e^-	Synch.	both	2×10^{-20}	10^{33}	2×10^{25}	CRAB	[1340,1341,1361]
e^-	VC	(+)	10^{-20}	10^{31}	10^{23}	CRAB	[1338,1344,1362]
γ	PD	(+)	7.1×10^{-19}	1.7×10^{33}	1.4×10^{24}	LH. J2032+4102	[1163]
γ	PD	(+)	1.3×10^{-17}	2.2×10^{31}	8×10^{22}	MultiSrc	[1356]
γ	PD	(+)	1.8×10^{-17}	1.4×10^{31}	5.8×10^{22}	eHWCJ1825−134	[1356]
γ	PD	(+)	2.2×10^{-17}	9.9×10^{30}	4.7×10^{22}	eHWCJ1907+063	[1356]
γ	3γ	(+)	–	–	2.5×10^{25}	LH. J2032+4102	[1163]
γ	3γ	(+)	–	–	1.2×10^{24}	eHWC J1825−134	[1356]
γ	3γ	(+)	–	–	1.0×10^{24}	eHWC J1907+063	[1356]
γ	3γ	(+)	–	–	4.1×10^{23}	CRAB	[1355]
γ	AS	(-)	–	–	1.7×10^{22}	diffuse (Tibet)	[1164]
γ	AS	(-)	–	–	6.8×10^{21}	LH. J1908+0621	[1164]
γ	AS	(-)	–	–	1.4×10^{21}	CRAB	[1355]
γ	AS	(-)	–	–	9.7×10^{20}	CRAB	[1355]
γ	AS	(-)	–	–	2.1×10^{20}	CRAB	[1361]
γ	PP	(-)	–	1.2×10^{29}	2.4×10^{21}	MultiSrc (6)	[1363]
γ	PP	(-)	2×10^{-16}	2.6×10^{28}	7.8×10^{20}	Mrk 501	[1348,1364]
γ	PP	(-)	–	1.9×10^{28}	3.1×10^{20}	MultiSrc (32)	[1359]

Fig.: Current constraints for LIV in QED. Abbr.: Synch. — synchrotron radiation, VC — vacuum Cherenkov radiation, PD — photon decay, 3γ — photon splitting, AS — air showers, PP — e^+e^-–produciton on extragalactic background light. See Addazi et al., 2022.

$n = 2$ LIV QED

The theory is:

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_e + \mathcal{L}_\gamma, \tag{3}$$

Higher dimension operators:

$$\mathcal{L}_\gamma = \frac{s_2}{4M_{\text{LIV}}^2} F_{kj} \partial_i^2 F^{kj}, \tag{4}$$

$$\mathcal{L}_e = i\kappa\bar{\psi}\gamma^i D_i \psi + \frac{i s_2^e}{M_{\text{LIV, e}}^2} D_j \bar{\psi}\gamma^i D_i D_j \psi, \tag{5}$$

where s_2, $s_2^e = \pm 1$.
$\mathcal{L}_\gamma$: $[s_2] \equiv [m]^0$, $[M_{\text{LIV}}] = [m]^1$.
$\mathcal{L}_e$: $[s_2^e] \equiv [m]^0$, $[\kappa] = [m]^0$, $[M_{\text{LIV, e}}] = [m]^1$.
M_{LIV} и $M_{\text{LIV, e}}$ — Lorentz invariance violation parameter.

LIV QED with $n = 2$: overview

We omit $\mathcal{L}_e$: constraints on $M_{\text{LIV},e} = 2 \times 10^{16}$ ГэВ (95% C.L.) from anomalous synchrotron radiation and vacuum Cherenkov radiation of soft electrons from Crab Nebula (arXiv:1207.0670 [gr-qc]).
Since that we consider only $\mathcal{L}_\gamma$, because the current constraint for «subluminal» theory is (arXiv:2106.06393 [hep-ph])

$$M_{\text{LIV}} > 1.7 \times 10^{13} \text{ ГэВ}. \tag{6}$$

The dispersion relation for «subluminal» photon reads as:

$$E_\gamma^2 = k_\gamma^2 - \frac{k_\gamma^4}{M_{\text{LIV}}^2}. \tag{7}$$

How to test the muon problem:

$$z \equiv \frac{\ln \left\langle N_{\mu}^{\text{obs}} \right\rangle - \ln \left\langle N_{\mu,p}^{\text{MC}} \right\rangle}{\ln \left\langle N_{\mu,Fe}^{\text{MC}} \right\rangle - \ln \left\langle N_{\mu,p}^{\text{MC}} \right\rangle}, \tag{8}$$

где $\left\langle N_{\mu}^{\text{obs}} \right\rangle$ — the average number of observed muons, $\left\langle N_{\mu,p}^{\text{MC}} \right\rangle$ ($\left\langle N_{\mu,Fe}^{\text{MC}} \right\rangle$) — average numbers of muons for proton(iron)-induced CR in MC.

If the EAS is initiated by a proton-CR, then $z = 0$; if it is ititiated by an iron-CR, then $z = 1$.

«The muon puzzle»

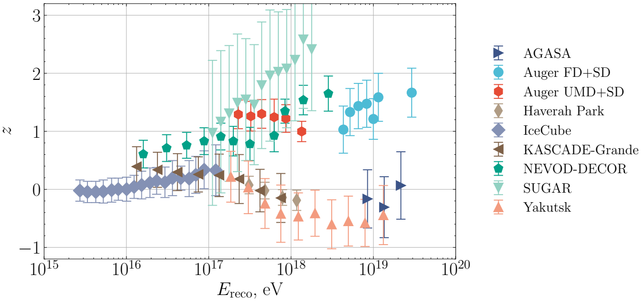


Fig.: Predictions for z-scale parameter based on muon densities (hadron model is EPOS-LHC). WHISP, 2023.

Who observes the muon anomaly: SUGAR, NEVOD-DECOR, Auger FD+SD, Auger UMD+SD.

The Bethe-Heitler process is crucial for the EAS evolution:

$$Z\gamma^*\gamma \to Ze^+e^- \tag{9}$$

— pair-production by an external photon γ in a Coloumb field of an atom (γ^*). The classical result is (H. Bethe, W. Heitler, 1934):

$$\sigma_{\mathrm{BH}}^{\mathrm{LI}} = \frac{28Z^2\alpha^3}{9m_e^2}\left[\log\frac{183}{Z^{1/3}} - \frac{1}{42}\right]. \tag{10}$$

Comparing the LI case with the process in the LIV case with $n = 2$ (G. Rubtsov, P. Satunin, S. Sibiryakov, 2012):

$$\frac{\sigma_{\mathrm{BH}}^{\mathrm{LIV}}}{\sigma_{\mathrm{BH}}^{\mathrm{LI}}} \simeq \frac{12m_e^2 M_{\mathrm{LIV}}^2}{7E_\gamma^4} \times \log\frac{E_\gamma^4}{2m_e^2 M_{\mathrm{LIV}}^2}. \tag{11}$$

Note that

$$\frac{\sigma_{\mathrm{BH}}^{\mathrm{LIV}}}{\sigma_{\mathrm{BH}}^{\mathrm{LI}}} \simeq \frac{12 m_e^2 M_{\mathrm{LIV}}^2}{7 E_\gamma^4} \times \log \frac{E_\gamma^4}{2 m_e^2 M_{\mathrm{LIV}}^2} \sim E_\gamma^{-4} \log E_\gamma^4, \tag{12}$$

therefore,

$$\sigma_{\mathrm{BH}}^{\mathrm{LIV}} \ll \sigma_{\mathrm{BH}}^{\mathrm{LI}}, \tag{13}$$

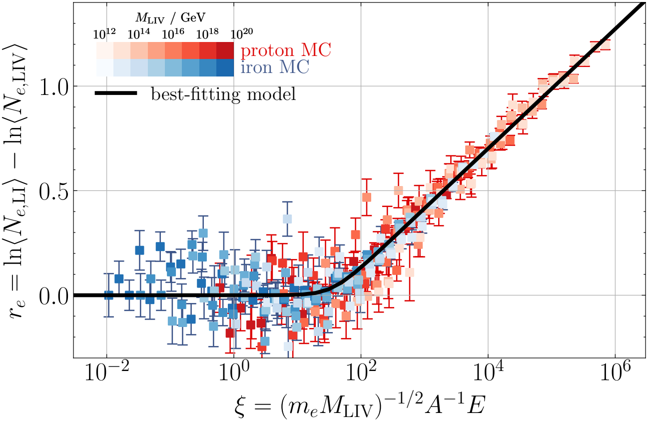
so for high-energies $\lambda^{\mathrm{LIV}} \gg \lambda^{\mathrm{LI}}$. This allows us to conclude that the development of the electron-positron shower will be reduced in the case of Lorentz violation (it will be less «intense»).

«The muon puzzle»

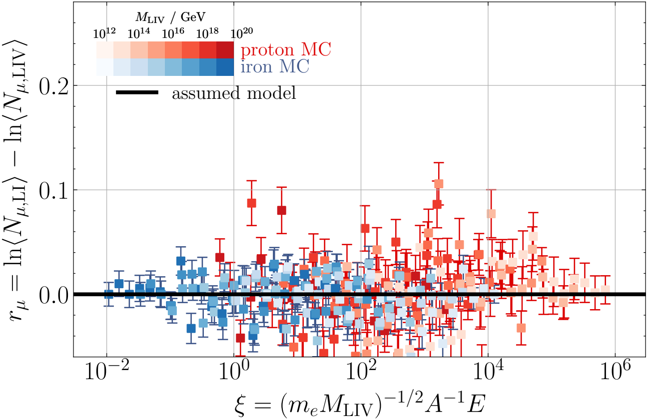
Analyzing the full development of the EAS, we find that the number of electrons satisfies $\langle N_{\mathrm{LIV},e} \rangle < \langle N_{\mathrm{LI},e} \rangle$. However, the number of muons remains unchanged (this requires verification through simulations). As a result, the energy of the primary particle is underestimated. Consequently, the expected number of muons is also underestimated due to the electron deficit, which leads to larger values of z.

Monte-Carlo, CORSIKA: $r_e(\xi)$

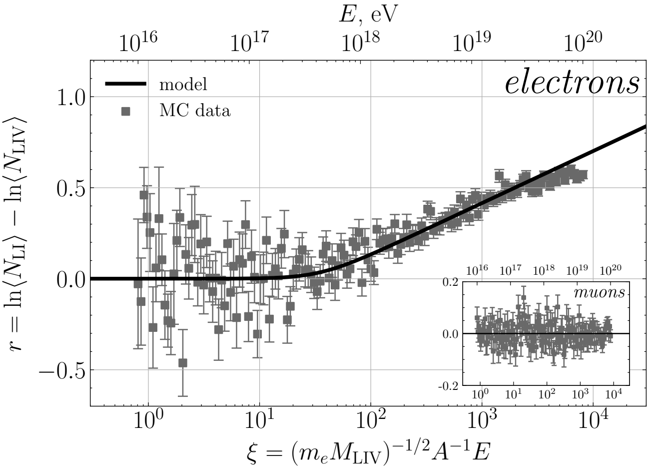
Первый шаг:



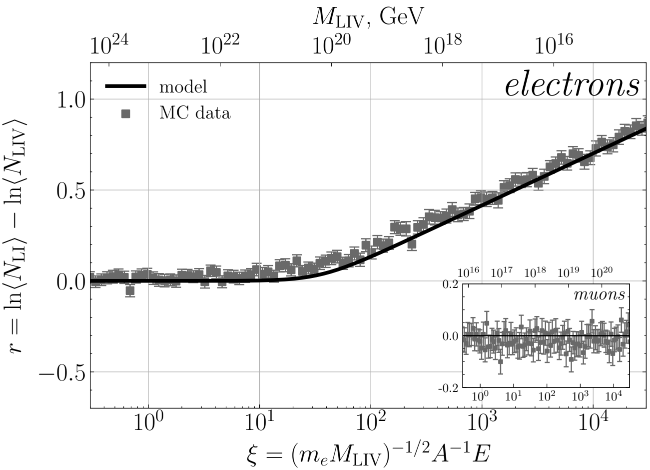
Monte-Carlo, CORSIKA: $r_\mu(\xi)$



Monte-Carlo, CORSIKA: fixed $M_{\mathrm{LIV}} = 3 \times 10^{17}$ GeV



Monte-Carlo, CORSIKA: fixed energy $E = 10^{19}$ eV



Results

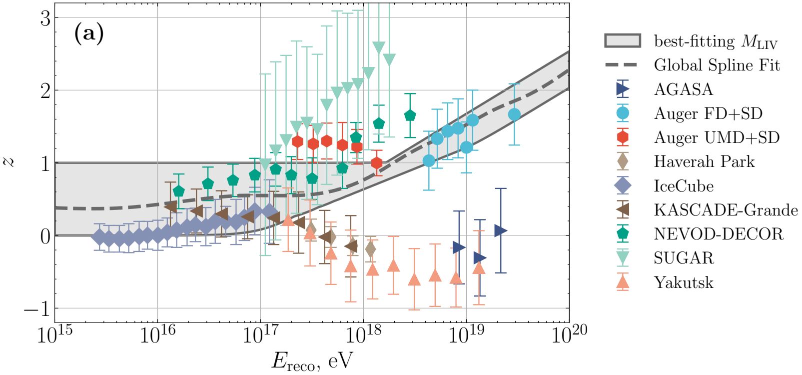


Fig.: Best-fit value is $M_{\text{LIV}} = 1.9 \times 10^{16}$ GeV based on Pierre Auger data.

Is there anything else?

Lorentz invariance violation can also be invoked to explain other phenomenological features at high energies.
Could the second «knee» (a spectral break) observed, for instance, in the Tunka-133 experiment be explained within this framework? (L. A. Kuzmichev.)

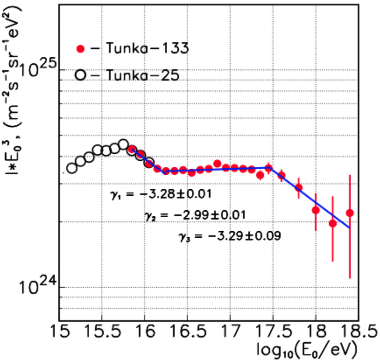
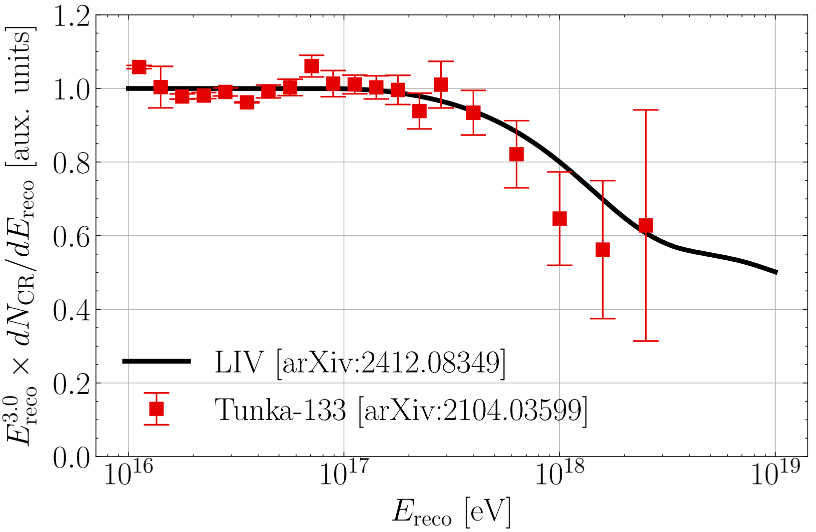


Fig.: The spectrum becomes steeper at energies $E \gtrsim 3 \times 10^{17}$ eV. arXiv:2104.03599

Tunka-133



1.2
1.0
0.8
0.6
0.4
0.2
0.0
$E^{3.0}_{\rm reco} \times dN_{\rm CR}/dE_{\rm reco}$ [aux. units]
10^{16}
10^{17}
10^{18}
10^{19}
$E_{\rm reco}$ [eV]
LIV [arXiv:2412.08349]
Tunka-133 [arXiv:2104.03599]

Conclusions

- We obtain a new constraint on the LIV QED parameter for $n = 2$, based on data from the Pierre Auger Observatory.
- For instance, our best-fit value, $M_{\text{LIV}} \sim 10^{16}$ GeV (95% C.L.), is preferred with generic LIV scenarios, including Horava–Lifshitz gravity.
- A broad field for phenomenological studies!

Дополнительные слайды

Различные модели адронных взаимодействий

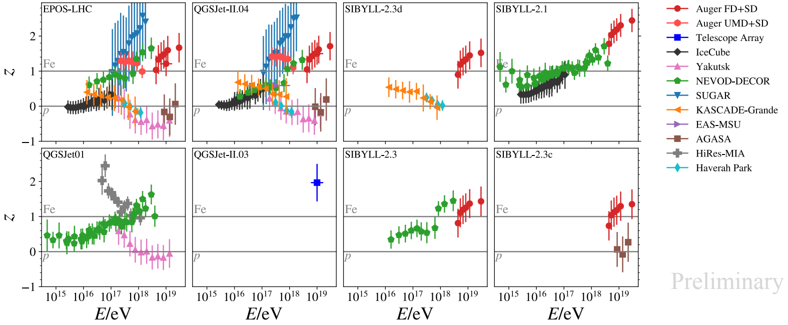


Fig.: Параметр z в различных адронных моделях. WHISP, 2023.

Лоренц-нарушение для других типов полей

Скалярное поле:

$$\mathcal{L}_s = i\frac{\kappa}{M_{\rm Pl}}\bar{\varphi}(n\cdot\partial)^3\varphi \implies E^2 \simeq \mathbf{p}^2 + m^2 + \frac{\kappa}{M_{\rm Pl}}|\mathbf{p}|^3. \qquad (14)$$

Фермионное поле:

$$\mathcal{L}_f = \frac{1}{M_{\rm Pl}}\bar{\psi}(\eta_1\gamma^\mu n_\mu + \eta_2\gamma^\mu n_\mu\gamma_5)(n\cdot\partial)^2\psi. \qquad (15)$$

Векторное поле — в докладе и в самой работе.

arXiv: 0301124 [hep-ph]

Число электронов можно оценить степенной функцией:

$$\langle N_e \rangle \propto A_{\mathrm{reco}}^{-\alpha_e} E_{\mathrm{reco}}^{\beta_e}, \tag{16}$$

то есть

$$\ln[E_{\mathrm{reco}}/\mathrm{GeV}] = \varepsilon_e + (\alpha_e \beta_e^{-1}) \ln A_{\mathrm{reco}} + \beta_e^{-1} \ln \langle N_e \rangle \,, \tag{17}$$

откуда видно, что мы действительно будем недооценивать энергию E_{reco}, так как $\langle N_{\mathrm{LIV},e} \rangle < \langle N_{\mathrm{LI},e} \rangle$.
Аналогично:

$$N_\mu \propto A^{\alpha_\mu} E^{\beta_\mu} \tag{18}$$

или

$$\ln \langle N_\mu \rangle = -n_\mu + \alpha_\mu \ln A + \beta_\mu \ln[E/\mathrm{GeV}]. \tag{19}$$

Развитие ШАЛ

Итого:

$$z = \frac{\ln A}{\ln 56} + \frac{\beta_\mu}{\alpha_\mu \ln 56} \ln \left[\frac{E}{E_{\text{reco}}} \right]. \tag{20}$$

Удобно ввести следующие параметры для анализа:

$$r_e \equiv \ln \left[\frac{\langle N_{e,\text{LI}} \rangle}{\langle N_{e,\text{LIV}} \rangle} \right], \quad r_\mu \equiv \ln \left[\frac{\langle N_{\mu,\text{LI}} \rangle}{\langle N_{\mu,\text{LIV}} \rangle} \right]. \tag{21}$$

«Универсальный» параметр:

$$\xi \equiv (m_e M_{\text{LIV}})^{-1/2} A^{-1} E. \tag{22}$$

При этом удобно параметризовать $r_e(\xi)$ следующим образом:

$$r_e(\xi) = r_{e,0} \ln \left[1 + \left(\frac{\xi}{\xi_0} \right)^{\varrho} \right], \quad r_\mu(\xi) = 0. \tag{23}$$

Симуляции на CORSIKA 7.7550

EPOS 1.99 (UrQMD 1.3.1) модель для высокоэнергетических (низкоэнергетических) адронных взаимодействий и EGS4 для электромагнитных взаимодействий.
Предполагается рассматривать только вертикальные ШАЛ $\theta = 0$ с анализом состава частиц на уровне моря. Симуляции проводятся для двух типов частиц: протонов и железа.
Энергеический порог на электроны: $E_e > 1$ МэВ, на мюоны: $E_\mu > 1$ ГэВ.

Алгоритм

1. Нахождение ε_e, α_e, β_e и n_μ, α_μ, β_μ в диапазоне энергий 10^{16} эВ до 5×10^{19} эВ в случае ЛИ. Усреднение по числу ливней.

параметры	ε_e	$\alpha_e \beta_e^{-1}$	β_e^{-1}	n_μ	α_μ	β_μ
значение	3.832	0.089	0.890	3.621	0.076	0.921
1σ	1.169	0.046	0.053	0.173	0.012	0.008

2. Модификация EGS4 в CORSIKA в диапазоне $M_{\text{LIV}} \in \{10^{13}, 10^{14}, \ldots, 10^{18}\}$ эВ. Аппроксимируем зависимости $r_e(\xi)$. Проверяем, что $r_\mu(\xi) = 0$.

значение	$r_{e,0}$	ξ_0	ϱ
best-fitting value	0.052	35.290	2.407
1σ	0.013	2.156	0.568